

Taylor

$$\begin{cases} y' = -y + x + 2 \\ y(0) = 2 \Rightarrow y_0 = 2 \\ 0 \leq x \leq 0,3 \Rightarrow h = 0,1 \end{cases} \quad \begin{aligned} y' = f &= -y + x + 2 \\ f' &= -1y' + 1 = -1(-y + x + 2) + 1 = y - x - 1 \\ f'' &= y' - 1 = (-y + x + 2) - 1 = -y + x + 1 \end{aligned}$$

$$\begin{aligned} f_0 &= f(x_0, y_0) = -y_0 + x_0 + 2 \Rightarrow f(0, 2) = -2 + 0 + 2 = 0 \\ f'_0 &= f'(x_0, y_0) = y_0 - x_0 - 1 \Rightarrow f'_0(0, 2) = 2 - 0 - 1 = 1 \\ f''_0 &= f''(x_0, y_0) = -y_0 + x_0 + 1 \Rightarrow f''_0(0, 2) = -2 + 0 + 1 = -1 \end{aligned}$$

$$x_1 = 0,1$$

$$y_1 = y_0 + h f_0 + \frac{h^2}{2!} f'_0 + \frac{h^3}{3!} f''_0 = 2 + (0,1)(0) + \frac{(0,1)^2}{2!}(1) + \frac{(0,1)^3}{3!}(-1) = 2,00483$$

$$\begin{aligned} f_1 &= f(x_1, y_1) = -y_1 + x_1 + 2 \Rightarrow f(0,1; 2,00483) = -2,00483 + 0,1 + 2 = 0,09517 \\ f'_1 &= f'(x_1, y_1) = y_1 - x_1 - 1 \Rightarrow f'_1(0,1; 2,00483) = 2,00483 - 0,1 - 1 = 0,90483 \\ f''_1 &= f''(x_1, y_1) = -y_1 + x_1 + 1 \Rightarrow f''_1(0,1; 2,00483) = -2,00483 + 0,1 + 1 = -0,90483 \end{aligned}$$

$$x_2 = 0,2$$

$$y_2 = y_1 + h f_1 + \frac{h^2}{2!} f'_1 + \frac{h^3}{3!} f''_1 = 2,00483 + (0,1)(0,09517) + \frac{(0,1)^2}{2!}(0,90483) + \frac{(0,1)^3}{3!}(-0,90483) = 2,01872$$

$$\begin{aligned} f_2 &= f(x_2, y_2) = -y_2 + x_2 + 2 \Rightarrow f(0,2; 2,01872) = -2,01872 + 0,2 + 2 = 0,18128 \\ f'_2 &= f'(x_2, y_2) = y_2 - x_2 - 1 \Rightarrow f'_2(0,2; 2,01872) = 2,01872 - 0,2 - 1 = 0,81872 \\ f''_2 &= f''(x_2, y_2) = -y_2 + x_2 + 1 \Rightarrow f''_2(0,2; 2,01872) = -2,01872 + 0,2 + 1 = -0,81872 \end{aligned}$$

$$x_3 = 0,3$$

$$y_3 = y_2 + h f_2 + \frac{h^2}{2!} f'_2 + \frac{h^3}{3!} f''_2 = 2,01872 + (0,1)(0,18128) + \frac{(0,1)^2}{2!}(0,81872) + \frac{(0,1)^3}{3!}(-0,81872) = 2,04081$$