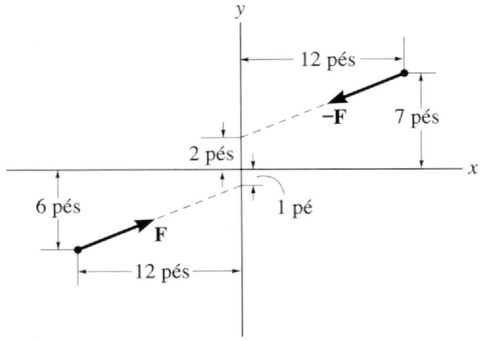
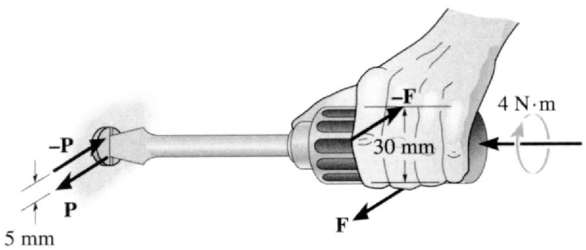
*4.72. Se o momento de binário tem intensidade de 300 lb · pés, determine a intensidade F das forças do binário.



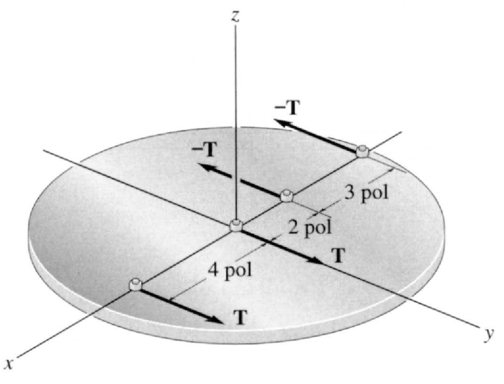
Problema 4.72

4.73. Um momento torsor de 4 N · m é aplicado ao cabo de uma chave de fenda. Decomponha esse momento de binário em um par de binários **F** exercido no cabo e **P** atuando na lâmina da ferramenta.



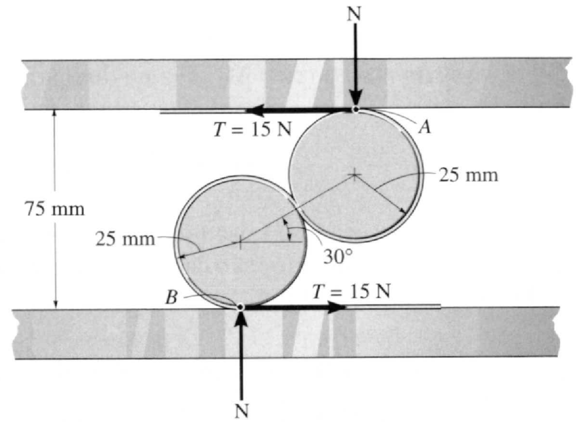
Problema 4.73

4.74. O momento de binário resultante criado pelos dois binários atuantes no disco é $\mathbf{M_R} = \{10\mathbf{k}\}$kip · pol. Determine a intensidade da força **T**.



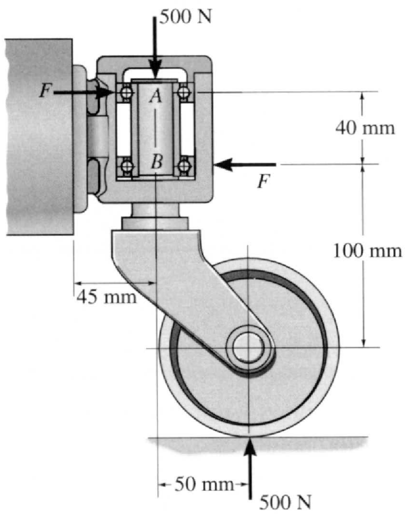
Problema 4.74

4.75. Um dispositivo chamado de 'rolamite' é empregado de várias maneiras para substituir movimento de escorregamento por movimento de rolagem. Se o cinto que envolve os roletes está sujeito à tensão de 15 N, determine as forças reativas N dos discos superior e inferior dos roletes, de modo que o binário resultante que atua neles seja nulo.



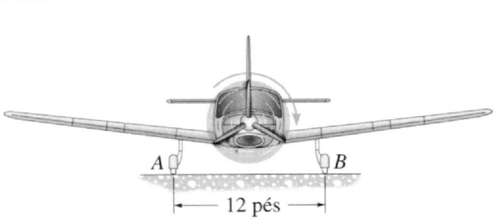
Problema 4.75

*4.76. O sistema de rodízio é submetido a dois binários. Determine as forças **F** que os dois mancais criam no eixo, de modo que o momento de binário resultante no rodízio seja nulo.



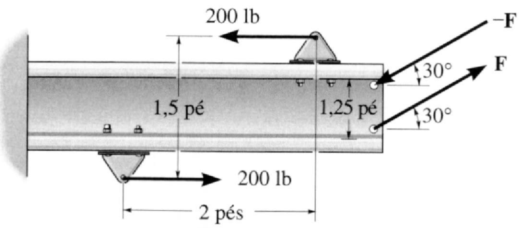
Problema 4.76

4.77. Quando o motor do avião está funcionando, a reação vertical que o solo exerce na roda em A é de 650 lb. Com o motor desligado, no entanto, as reações verticais em A e B são de 575 lb cada uma. A diferença nas leituras da força em A é provocada pela ação de um momento de binário nas hélices quando o motor está em funcionamento. Esse momento tende a tombar o avião no sentido anti-horário, que é oposto ao sentido horário de rotação das hélices. Determine a intensidade desse momento de binário e a intensidade da força de reação exercida em B quando o motor está em funcionamento.



Problema 4.77

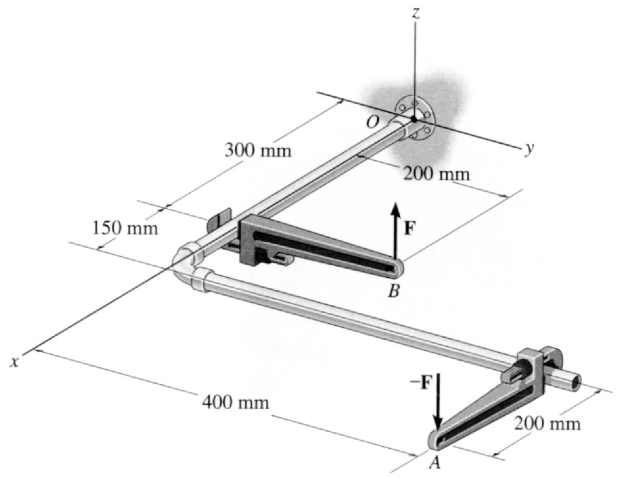
4.78. Dois binários atuam na viga. Determine a intensidade de **F**, de modo que o momento de binário resultante seja 450 lb · pés no sentido anti-horário. Em que local da viga o momento de binário resultante atua?



Problema 4.78

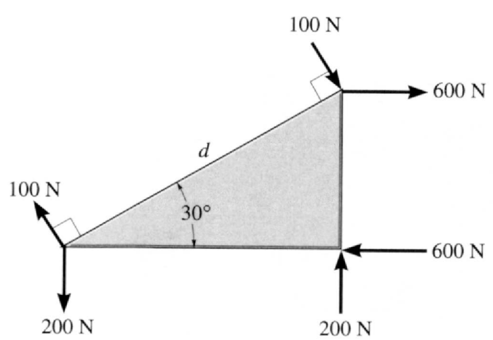
4.79. Expresse o momento de binário que atua na estrutura tubular na forma de vetor cartesiano. Resolva o problema (a) utilizando a Equação 4.13 e (b) somando os momentos de força em relação ao ponto *O*. Considere F = {25k}N.

***4.80.** Se o momento de binário atuando nos tubos tem intensidade de 400 N · m, determine a intensidade *F* da força vertical aplicada em cada chave.



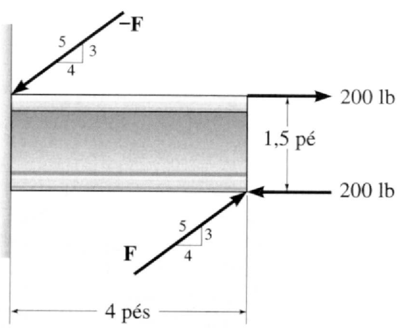
Problemas 4.79/80

4.81. As extremidades da chapa triangular estão sujeitas a três binários. Determine a dimensão *d* da chapa de modo que o momento de binário resultante seja 350 N · m no sentido horário.



Problema 4.81

4.82. Dois binários atuam na viga mostrada na figura. Determine a intensidade de **F** de modo que o momento de binário resultante seja 300 lb · pés no sentido anti-horário. Em que local da viga o momento de binário resultante atua?

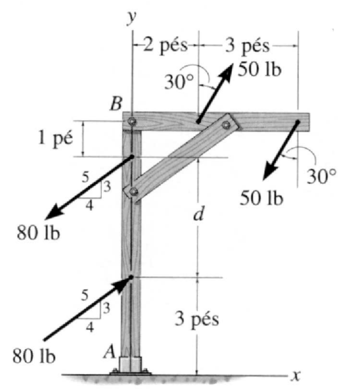


Problema 4.82

4.83. Dois binários atuam na estrutura da figura. Determine a distância *d* entre as forças do binário de 80 lb para que o momento de binário resultante seja nulo.

***4.84.** Dois binários atuam na estrutura. Se $d = 4$ pés, determine o momento de binário resultante. Calcule o mesmo resultado decompondo cada força nos componentes *x*, *y* e obtenha o momento de cada binário (a) por meio da Equação 4.13 e (b) somando os momentos de todos os componentes de força em relação ao ponto *A*.

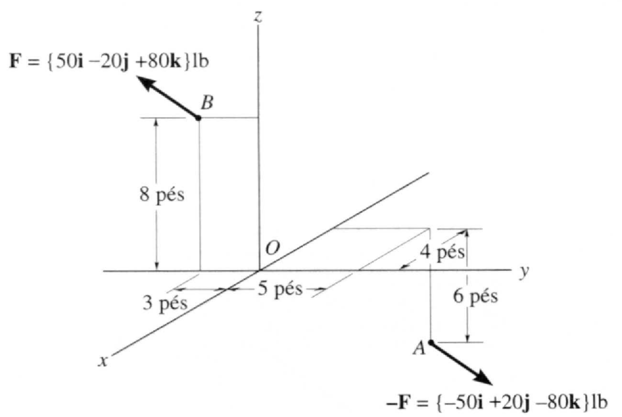
4.85. Dois binários atuam na estrutura. Se $d = 4$ pés, determine o momento de binário resultante. Calcule o mesmo resultado decompondo cada força nos componentes *x*, *y* e obtenha o momento de cada binário (a) por meio da Equação 4.13 e (b) somando os momentos de todos os componentes de força em relação ao ponto *B*.



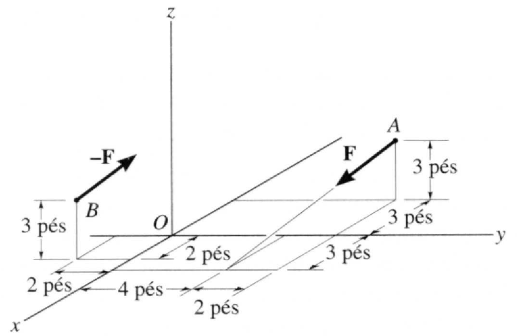
Problemas 4.83/84/85

4.86. Determine o momento de binário. Expresse o resultado como um vetor cartesiano.

4.87. Determine o momento de binário. Expresse o resultado como um vetor cartesiano. Cada força tem intensidade $F = 120$ lb.

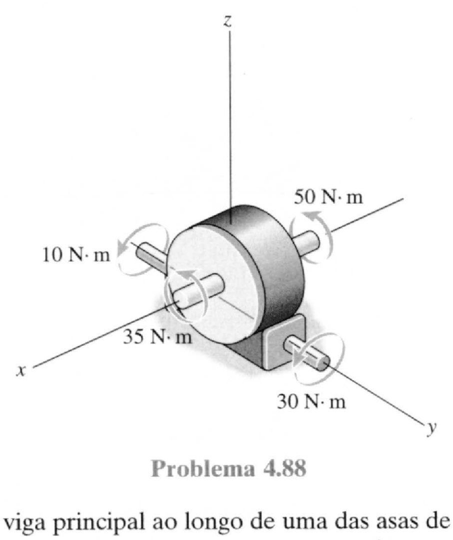


Problema 4.86



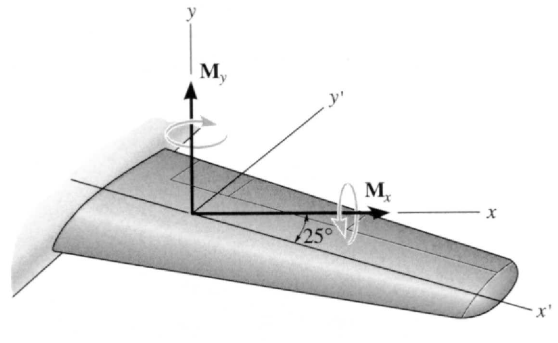
Problema 4.87

*4.88. O redutor de velocidade está sujeito a quatro momentos binários. Determine a intensidade do momento de binário resultante e seus ângulos diretores coordenados.



Problema 4.88

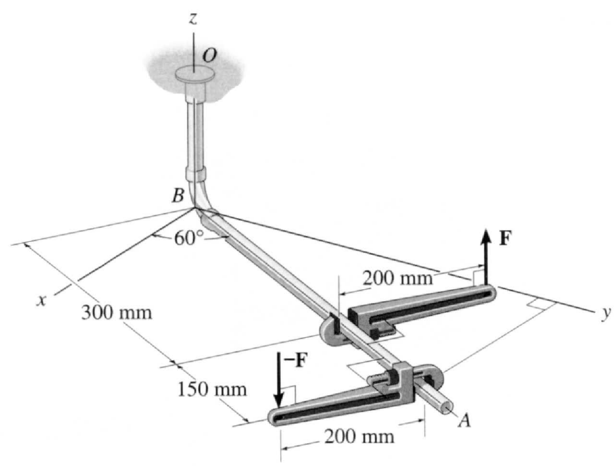
4.89. A viga principal ao longo de uma das asas de um aeroplano é forçada para trás a um ângulo de 25°, como mostra a figura. Com base nos cálculos das cargas que atuam sobre a asa foi determinado que a viga está sujeita aos momentos de binários M_x = 17 kip · pés e M_y = 25 kip · pés. Determine os momentos de binário resultantes gerados em relação aos eixos x' e y'. Os eixos se localizam no mesmo plano horizontal.



Problema 4.89

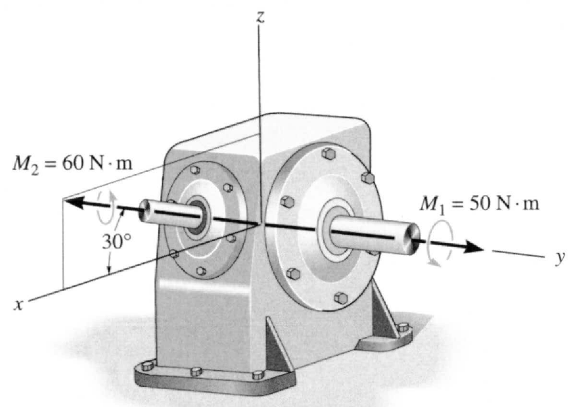
4.90. Como **F** = {100**k**} N, determine o momento de binário que atua na montagem. Expresse o resultado como um vetor cartesiano. O elemento BA está localizado no plano $x-y$.

4.91. Como a intensidade do momento de binário resultante é igual a 15 N · m, determine a intensidade F das forças aplicadas sobre as chaves.



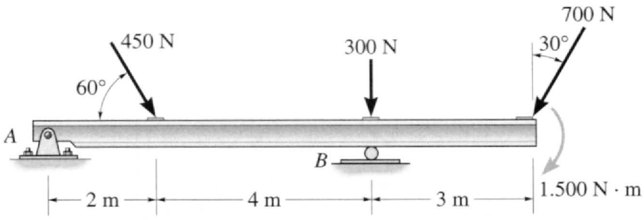
Problemas 4.90/91

*4.92. O redutor de velocidade está sujeito ao momento de binário mostrado na figura. Determine o momento de binário resultante, especificando sua intensidade e os ângulos diretores coordenados.



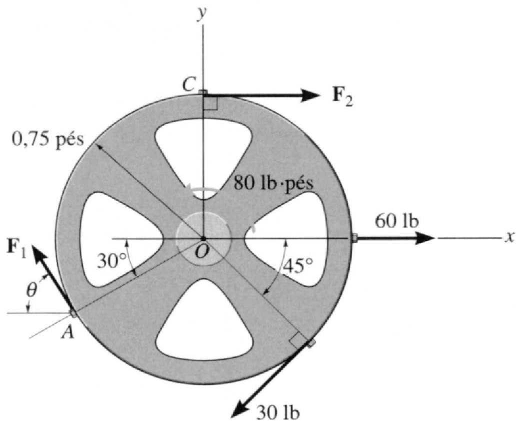
Problema 4.92

***4.116.** Substitua as cargas atuantes na viga por uma única força resultante. Especifique onde a força atua, tomando como referência o ponto *B*.



Problemas 4.115/116

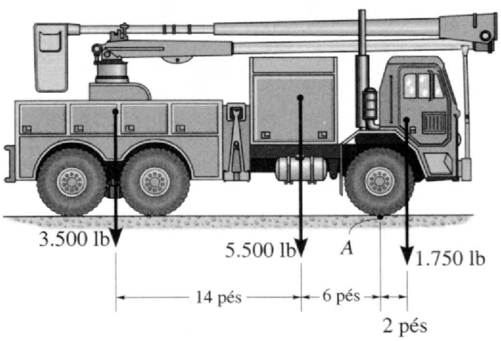
4.117. Determine as intensidades de $\mathbf{F}_1$ e $\mathbf{F}_2$ e a direção e sentido de $\mathbf{F}_1$ de forma que as cargas da figura produzam uma força e um momento resultante nulos sobre a roda.



Problema 4.117

4.118. Os pesos dos vários componentes do caminhão são mostrados na figura. Substitua esse sistema de forças por uma força resultante e um momento equivalentes com atuação no ponto *A*.

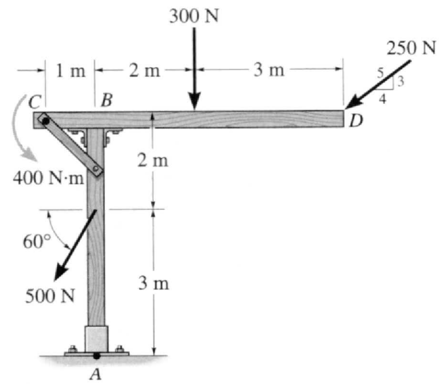
4.119. Agora, substitua esse sistema de forças por uma força resultante e especifique sua localização a partir do ponto *A*.



Problemas 4.118/119

***4.120.** Substitua as cargas sobre a estrutura por uma única força resultante. Especifique onde sua linha de ação intercepta o elemento *AB*, tomando como referência o ponto *A*.

4.121. Agora, especifique onde sua linha de ação intercepta o elemento *CD*, tomando como referência a extremidade *C*.

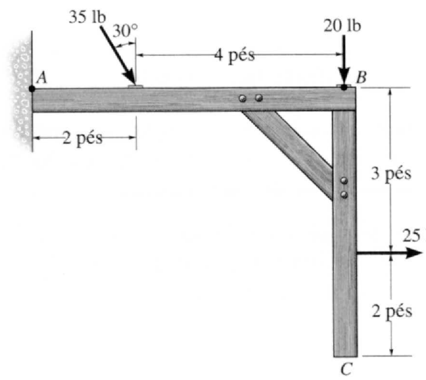


Problemas 4.120/121

4.122. Substitua o sistema de forças agindo na estrutura por uma força resultante equivalente e especifique onde a linha de ação da resultante intercepta o elemento *AB*, medido a partir do ponto *A*.

4.123. Substitua o sistema de forças atuantes na estrutura por uma força resultante equivalente e especifique onde a linha de ação da resultante intercepta o elemento *BC*, medido a partir do ponto *B*.

***4.124.** Substitua o sistema de forças atuantes sobre a estrutura por uma força resultante e um momento equivalentes ao sistema sobre o ponto *A*.

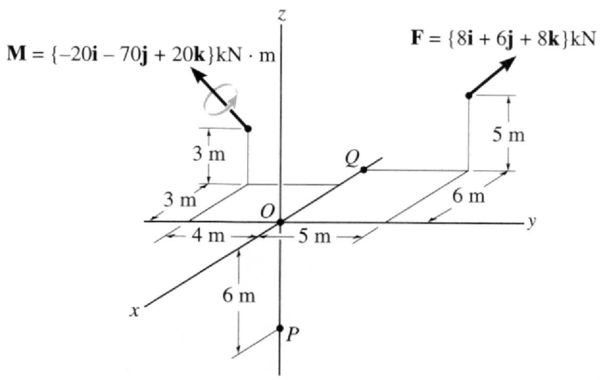


Problemas 4.122/123/124

4.125. Substitua o sistema de forças e de momentos binários por uma força resultante e um momento equivalente ao sistema no ponto *O*. Expresse os resultados na forma de vetores cartesianos.

4.126. Substitua o sistema de forças e de momentos binários por uma força resultante e um momento equivalente ao sistema no ponto *P*. Expresse os resultados na forma de vetores cartesianos.

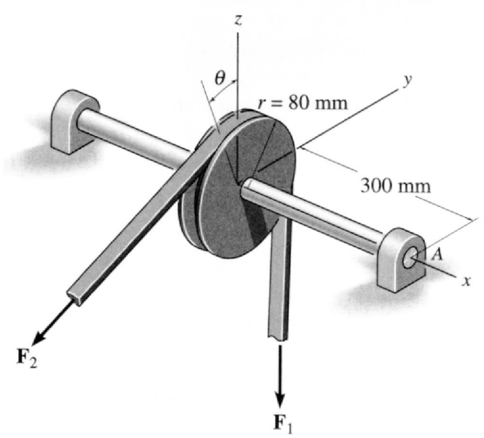
4.127. Substitua o sistema de forças e de momentos binários por uma força resultante e um momento equivalente ao sistema no ponto Q. Expresse os resultados na forma de vetores cartesianos.



Problemas 4.125/126/127

*__4.128.__ A correia que passa pela polia é submetida às forças $\mathbf{F}_1$ e $\mathbf{F}_2$, cada uma com intensidade de 40 N. A força $\mathbf{F}_1$ atua na direção $-\mathbf{k}$. Substitua essas forças por uma força e momento equivalentes no ponto A. Expresse o resultado na forma de vetor cartesiano. Considere $\theta = 0°$, de forma que $\mathbf{F}_2$ atue na direção $-\mathbf{j}$.

4.129. Agora, faça o mesmo, mas considere que $\theta = 45°$.

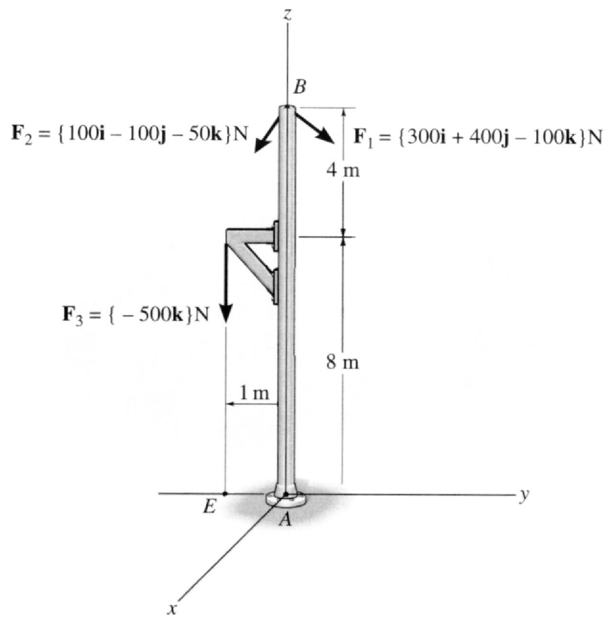


Problemas 4.128/129

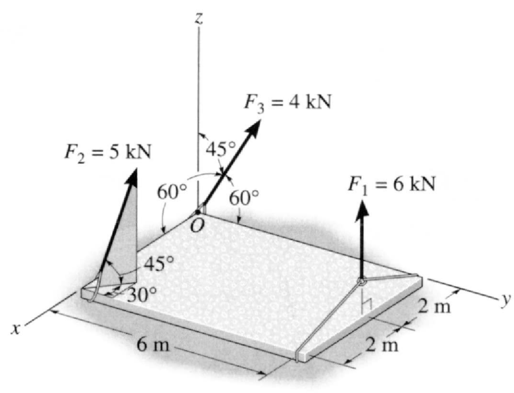
4.130. Substitua o sistema de forças por uma força e um momento equivalentes no ponto A.

4.131. A lâmina está prestes a ser içada pelos três cabos mostrados na figura. Substitua o sistema de forças que atua nos cabos por uma força e um momento equivalentes no ponto O. A força $\mathbf{F}_1$ é vertical.

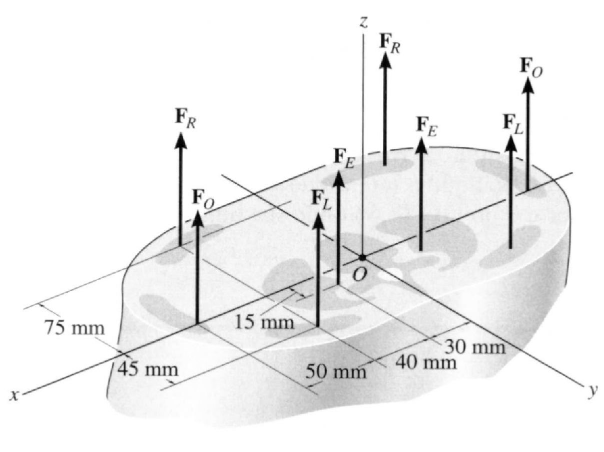
*__4.132.__ Um modelo biomecânico da região lombar do corpo humano é mostrado na figura. As forças que atuam nos quatro grupos musculares são: $F_R = 35$ N para o músculo reto abdominal, $F_O = 45$ N para o oblíquo abdominal, $F_L = 23$ N para o *lumbar latissimus dorsi* e $F_E = 32$ N para o *erector spinae*. Essas cargas são simétricas em relação ao plano $y-z$. Substitua esse sistema de forças paralelas por uma força e momento equivalentes ao sistema que atuam na espinha dorsal, no ponto O. Expresse os resultados na forma de vetores cartesianos.



Problema 4.130



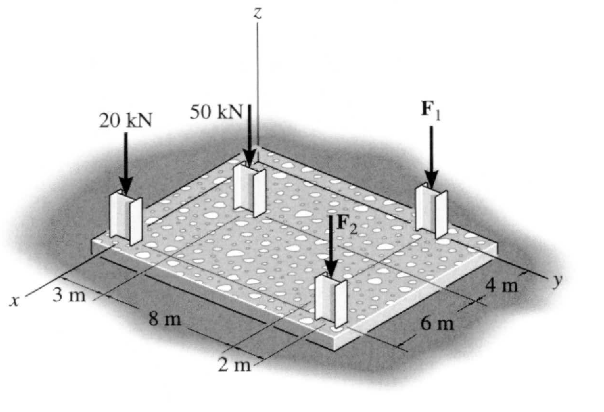
Problema 1.131



Problema 4.132

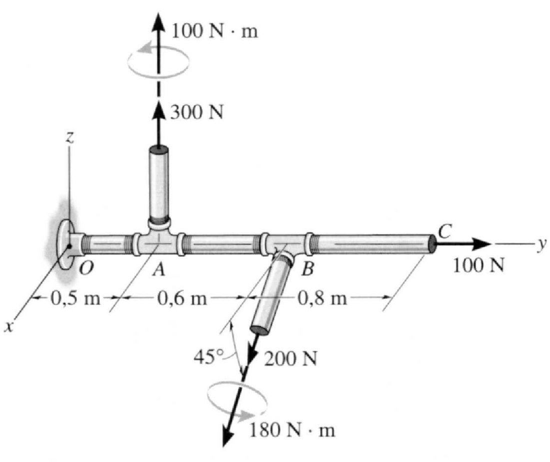
4.133. A laje da figura está submetida a quatro colunas paralelas com cargas. Determine a força resultante equivalente e especifique sua localização (x, y) sobre a laje. Considere que $F_1 = 30$ kN e $F_2 = 40$ kN.

4.134. Agora, faça o mesmo, mas considere que $F_1 = 20$ kN e $F_2 = 50$ kN.



Problemas 4.133/134

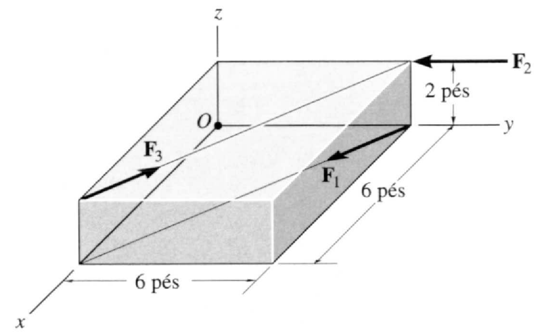
4.135. Substitua os dois torsores que atuam na estrutura de tubos por uma força resultante e um momento equivalentes no ponto O.



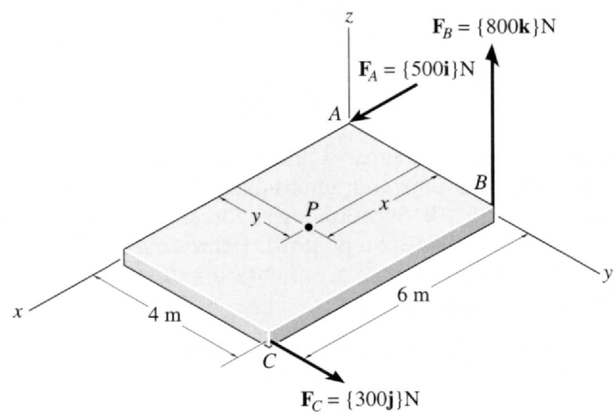
Problema 4.135

***4.136.** As três forças que atuam no bloco têm, cada uma intensidade de 10 lb. Substitua esse sistema por um torsor e especifique o ponto em que sua linha de ação intercepta o eixo z, tomando como referência o ponto O.

4.137. Substitua as três forças que atuam na placa por um torsor. Especifique a intensidade da força, o momento para o torsor e o ponto $P(x, y)$ onde sua linha de ação intercepta a placa.

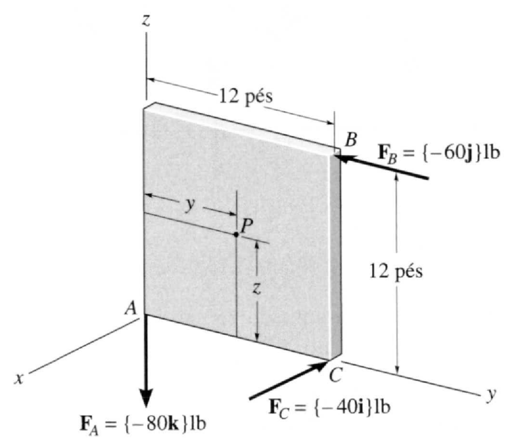


Problema 4.136



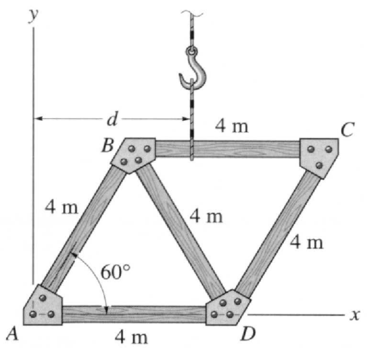
Problema 4.137

4.138. Substitua as três forças atuantes na placa por um torsor. Especifique a intensidade da força, o momento para o torsor e o ponto $P(y, z)$ em que sua linha de ação intercepta a placa.



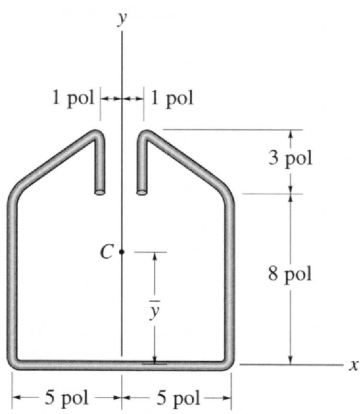
Problema 4.138

***9.48.** A treliça mostrada é feita de cinco elementos, cada um com comprimento de 4 m e massa por unidade de comprimento de 7 kg/m. Considerando as massas das placas de reforço nas juntas e as espessuras dos elementos como desprezíveis, determine a distância d até onde o cabo para elevação deve ser colocado, de forma que a treliça não se incline (gire) quando içada.



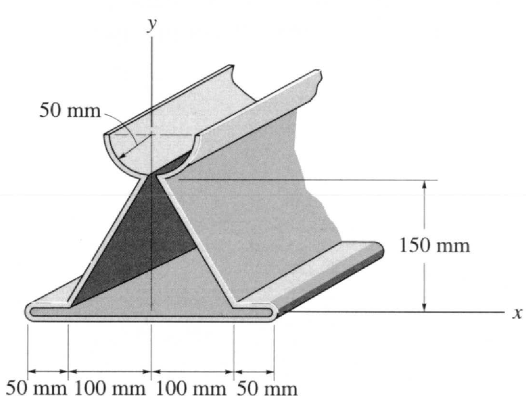
Problema 9.48

9.49. Localize o centróide para o fio dobrado. Despreze a espessura e pequenas deformações nas quinas do material.



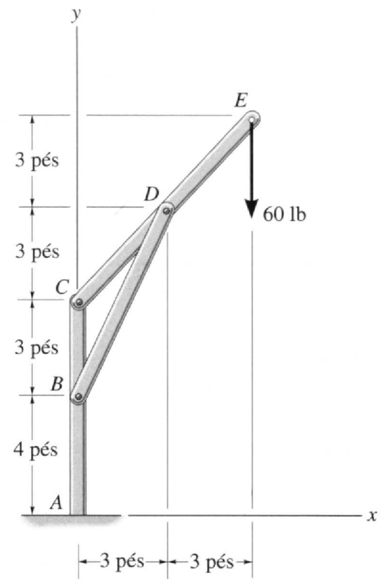
Problema 9.49

9.50. Localize o centróide $(\bar{x}, \bar{y})$ da seção transversal do metal. Despreze a espessura e pequenas deformações nas quinas do material.



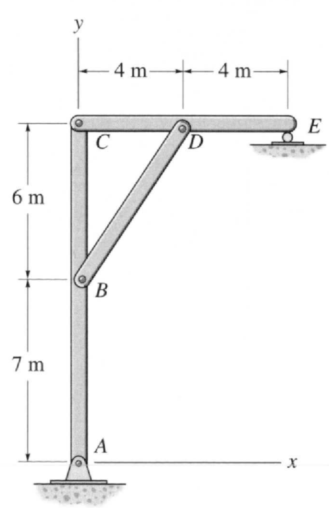
Problema 9.50

9.51. Os três elementos da estrutura têm peso por unidade de comprimento de 4 lb/pé cada um. Localize a posição $(\bar{x}, \bar{y})$ do centro de gravidade da estrutura. Despreze as dimensões dos pinos nas juntas e a espessura dos elementos. Calcule também as reações no apoio fixo A.



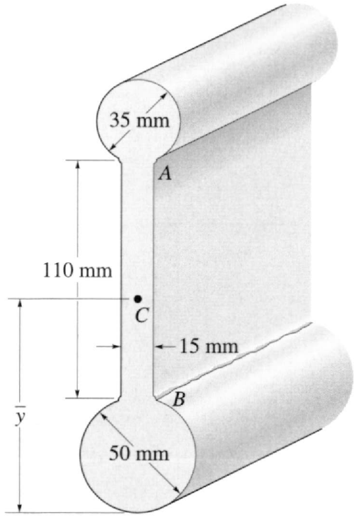
Problema 9.51

***9.52.** Cada um dos três elementos da estrutura tem massa por unidade de comprimento de 6 kg/m. Localize a posição $(\bar{x}, \bar{y})$ do centro de gravidade. Despreze as dimensões dos pinos nas juntas e a espessura dos elementos. Calcule também as reações no pino A e no rolete E.



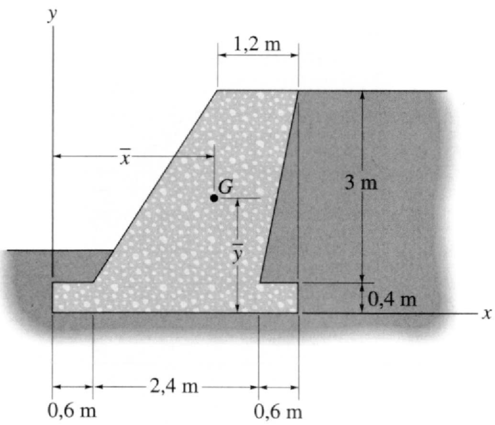
Problema 9.52

9.53. Determine a localização $\bar{y}$ do centróide da área da seção reta da viga. Despreze as dimensões das soldas das quinas em A e B para esses cálculos.



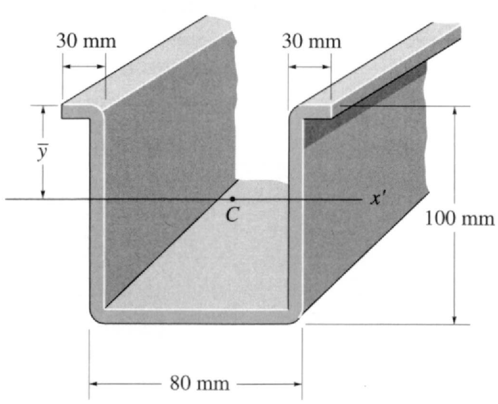
Problema 9.53

9.54. A barragem de gravidade é feita de concreto. Determine a localização ($\bar{x}$, $\bar{y}$) do centro de gravidade G para a parede.



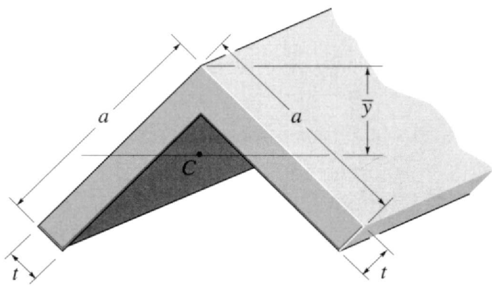
Problema 9.54

9.55. Um pontalete de alumínio tem seção transversal conhecida como chapéu fundo. Localize o centróide $\bar{y}$ de sua área. Cada parte constituinte tem espessura de 10 mm.



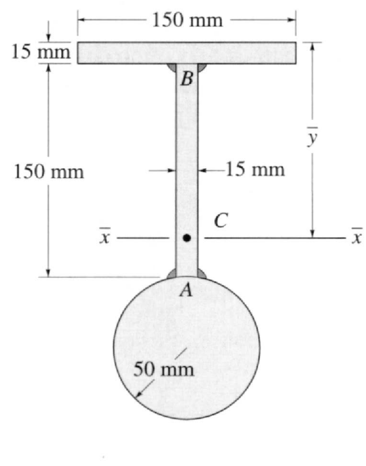
Problema 9.55

***9.56.** Localize o centróide $\bar{y}$ para a área da seção reta do perfil em ângulo.



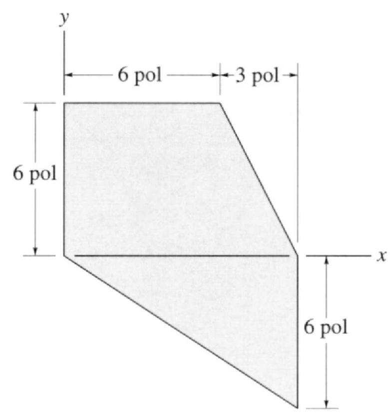
Problema 9.56

9.57. Determine a localização $\bar{y}$ do eixo $\bar{x}$ $\bar{x}$ do centróide da área da seção transversal da viga. Despreze as dimensões das soldas nas quinas em A e B para esses cálculos.



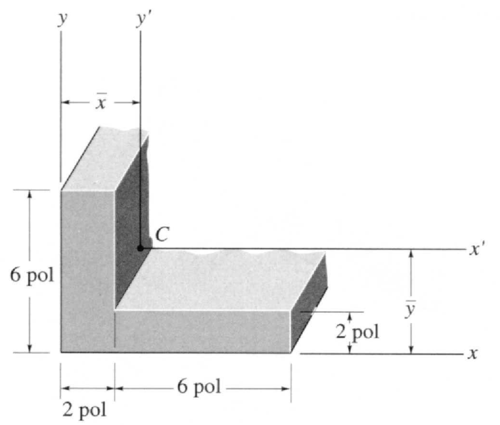
Problema 9.57

9.58. Determine a localização ($\bar{x}$, $\bar{y}$) do centróide C da área da figura.



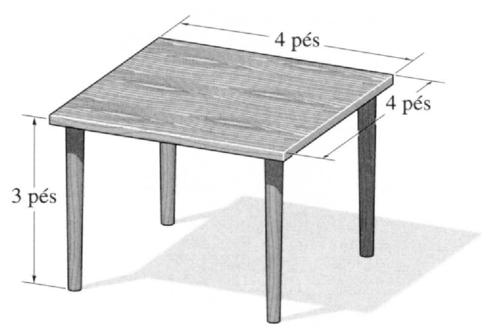
Problema 9.58

9.59. Localize o centróide ($\bar{x}$, $\bar{y}$) para a área da seção reta do perfil em ângulo.



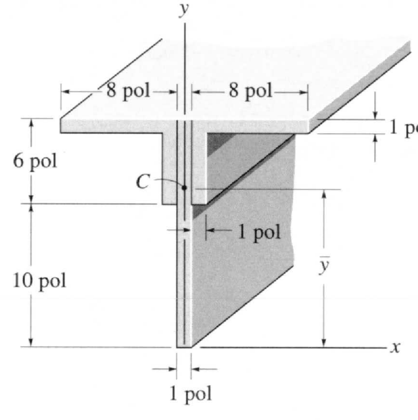
Problema 9.59

*9.60. A mesa de madeira é feita de uma tábua quadrada que tem peso de 15 lb. Cada uma das pernas pesa 2 lb e tem 3 pés de comprimento. Determine a que distância do solo está seu centro de gravidade. Qual é o ângulo, medido em relação à horizontal, em que o tampo da mesa pode ser inclinado sobre duas de suas pernas antes que ela tombe? Despreze a espessura de cada perna.



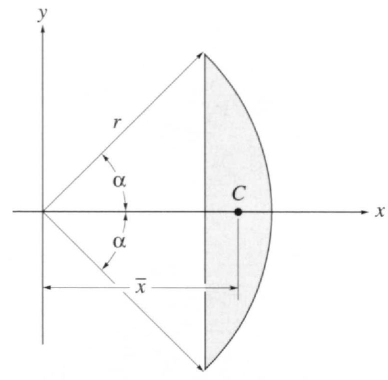
Problema 9.60

9.61. Localize o centróide $\overline{y}$ da área da seção reta da viga.



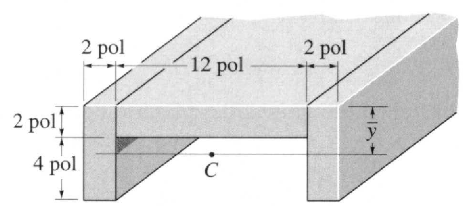
Problema 9.61

9.62. Determine a localização $\overline{x}$ do centróide C da área sombreada, que é parte de um círculo com raio r.



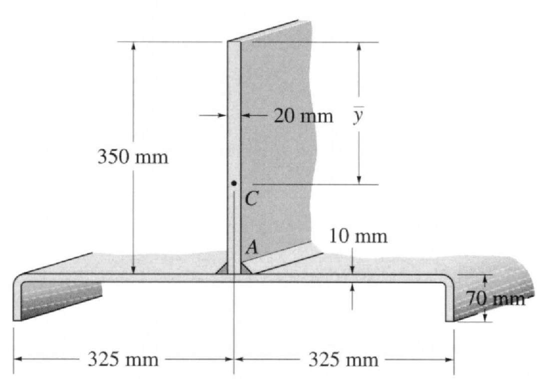
Problema 9.62

9.63. Localize o centróide $\overline{y}$ da área de seção reta do perfil.



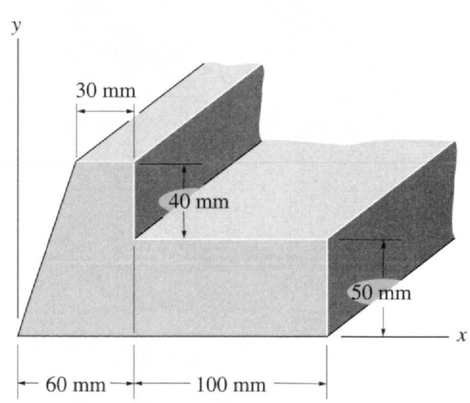
Problema 9.63

*9.64. Localize o centróide $\overline{y}$ da área da seção transversal da viga construída com um perfil e uma chapa. Suponha que todas as quinas sejam quadradas e despreze a dimensão da solda em A.



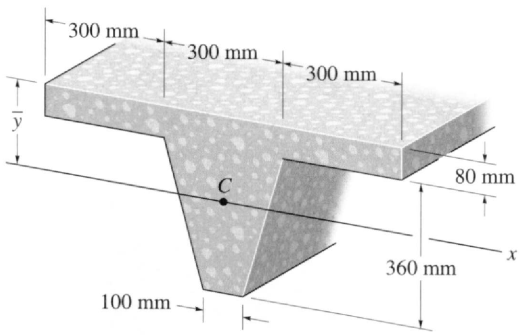
Problema 9.64

9.65. Localize o centróide $(\overline{x}, \overline{y})$ da área de seção reta do elemento.



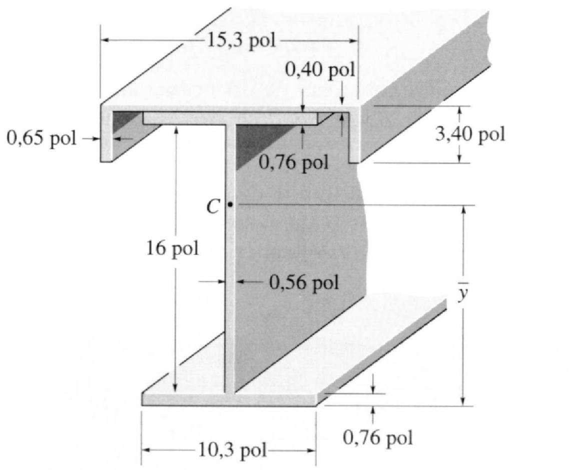
Problema 9.65

9.66. Localize o centróide $\overline{y}$ da viga de concreto com seção transversal afilada, como mostrado na figura.



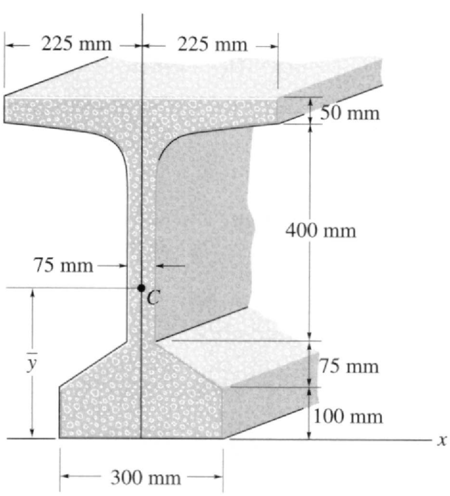
Problema 9.66

9.67. Localize o centróide $\overline{y}$ da seção transversal da viga composta de um perfil e uma viga de abas largas.



Problema 9.67

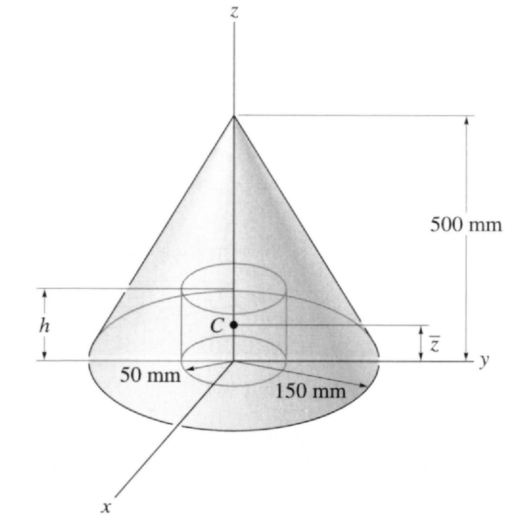
9.68. Localize o centróide $\overline{y}$ da seção transversal do pedestal.



Problema 9.68

9.69. Determine a distância h de um furo com diâmetro de 100 mm que deve ser perfurado na base de um cone para que o centro de massa do objeto resultante seja localizado em $\overline{z} = 115$ mm. O material tem densidade de 8 t/m^3.

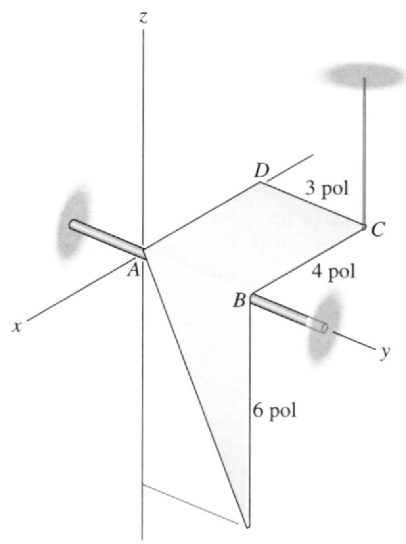
9.70. Determine a distância $\overline{z}$ do centróide do objeto que consiste em um cone com um furo de altura $h = 50$ mm perfurado na sua base.



Problemas 9.69/70

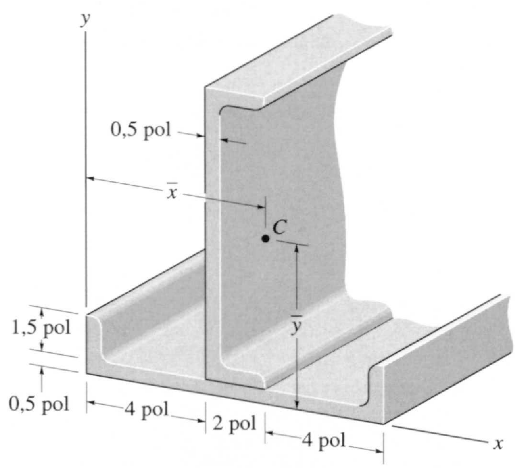
9.71. A peça de metal laminado tem as dimensões mostradas na figura. Determine a localização $(\overline{x}, \overline{y}, \overline{z})$ de seu centróide.

***9.72.** A peça de metal laminado tem peso por unidade de área de 2 lb/pé^2 e é sustentado por uma barra lisa e por uma corda em C. Se a corda for cortada, a peça vai sofrer uma rotação em torno do eixo y até atingir o equilíbrio. Determine o ângulo de inclinação na condição de equilíbrio, medido para baixo a partir do eixo negativo x, que AD forma com o eixo $-x$.



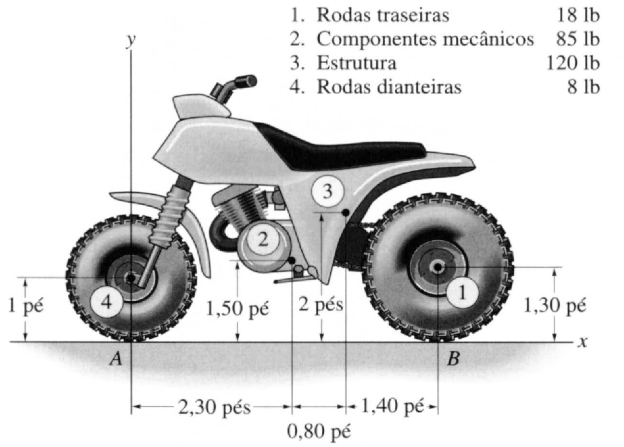
Problemas 9.71/72

9.73. Determine a localização $(\overline{x}, \overline{y})$ do centróide C da área da seção transversal do elemento estrutural construído de dois perfis de mesmas dimensões, soldados entre si como mostra a figura. Considere que todas as quinas são quadradas. Despreze as dimensões das soldas.



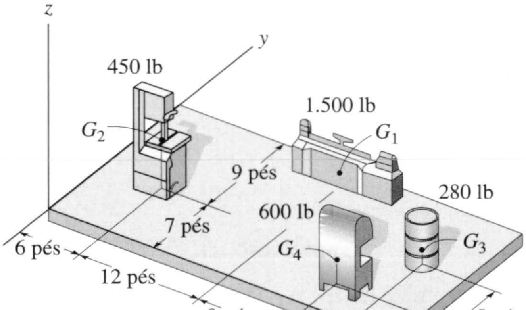
Problema 9.73

9.74. Determine a localização ($\overline{x}$, $\overline{y}$) do centro de gravidade do triciclo. As localizações dos centros de gravidade e os pesos de cada componente aparecem tabelados na figura. Se o triciclo é simétrico em relação ao plano $x-y$, determine as reações normais que cada uma de suas rodas exerce no solo.



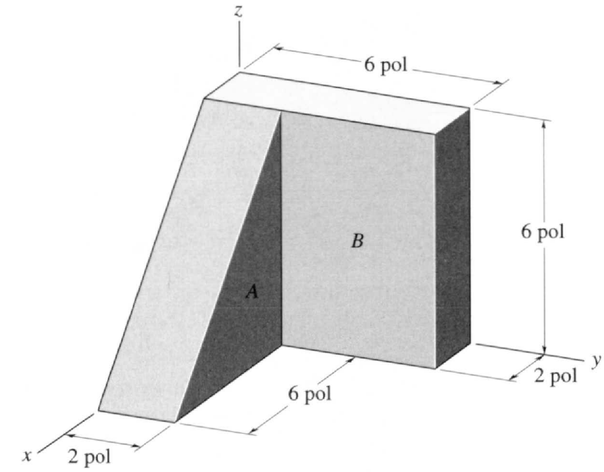
Problema 9.74

9.75. A maior parte da carga sobre o piso de um centro de compras é causada pelos pesos dos objetos mostrados na figura. Cada força atua através de seus respectivos centros de gravidade G. Localize o centro de gravidade ($\overline{x}$, $\overline{y}$) de todos esses componentes.



Problema 9.75

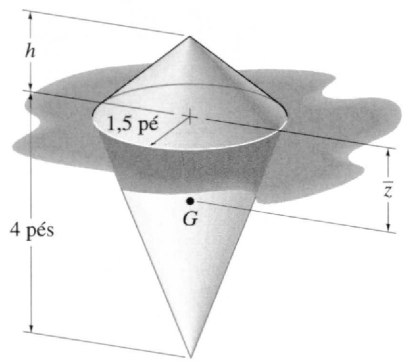
***9.76.** Localize o centro de gravidade do conjunto de dois blocos. Os pesos específicos dos materiais A e B são $\gamma_A = 150$ lb/pé^3 e $\gamma_B = 400$ lb/pé^3, respectivamente.



Problema 9.76

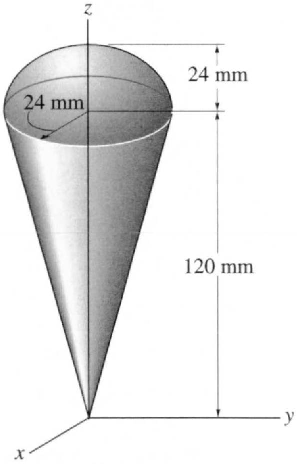
9.77. A bóia é composta de dois cones homogêneos, cada qual com raio de 1,5 pé. Sendo h = 1,2 pé, encontre a distância $\overline{z}$ para o centro de gravidade da bóia.

9.78. Sendo necessário que o centro de gravidade da bóia do problema anterior esteja localizado em $\overline{z}$ = 0,5 pé, determine a altura h do vértice superior do cone.



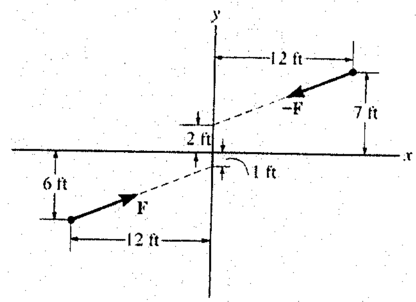
Problemas 9.77/78

9.79. Localize o centróide $\overline{z}$ do pião composto de um hemisfério e um cone.



Problema 9.79

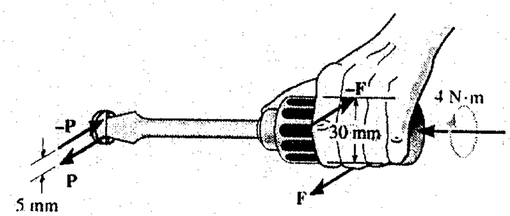
***4-72.** If the couple moment has a magnitude of 300 lb · ft, determine the magnitude F of the couple forces.



$$300 = F\left(\frac{12}{13}\right)(13) - F\left(\frac{5}{13}\right)(24)$$

$F = 108$ lb **Ans**

4-73. A twist of 4 N · m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



For the handle

$M_C = \Sigma M_x;$ $F(0.03) = 4$

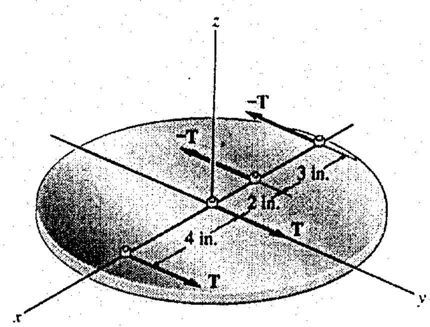
$F = 133$ N **Ans**

For the blade,

$M_C = \Sigma M_x;$ $P(0.005) = 4$

$P = 800$ N **Ans**

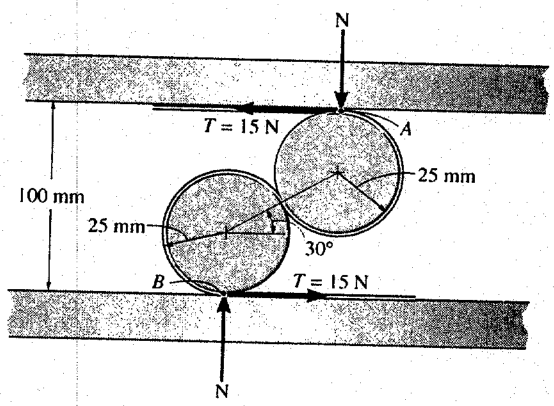
4-74. The resultant couple moment created by the two couples acting on the disk is $\mathbf{M_R} = \{10\mathbf{k}\}$kip · in. Determine the magnitude of force **T**.



$M_R = \Sigma M_z;$ $10 = T(9) + T(2)$

$T = 0.909$ kip **Ans**

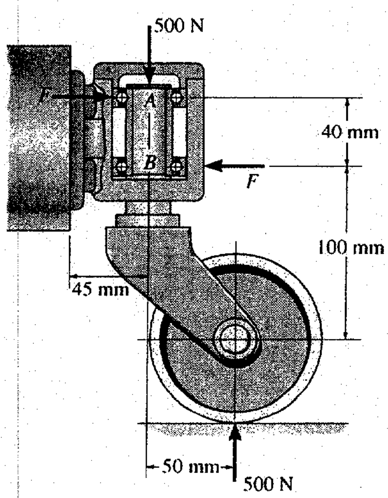
4-75. A device called a rolamite is used in various ways to replace slipping motion with rolling motion. If the belt, which wraps between the rollers, is subjected to a tension of 15 N, determine the reactive forces N of the top and bottom plates on the rollers so that the resultant couple acting on the rollers is equal to zero.



$\circlearrowleft + \Sigma M_A = 0; \quad 15(50 + 50\sin 30°) - N(50\cos 30°) = 0$

$N = 26.0 \text{ N}$ **Ans**

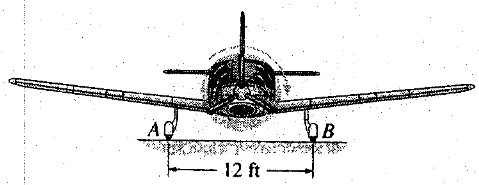
4-76. The caster wheel is subjected to the two couples. Determine the forces **F** that the bearings create on the shaft so that the resultant couple moment on the caster is zero.



$\circlearrowleft + \Sigma M_A = 0; \quad 500(50) - F(40) = 0$

$F = 625 \text{ N}$ **Ans**

4-77. When the engine of the plane is running, the vertical reaction that the ground exerts on the wheel at A is measured as 650 lb. When the engine is turned off, however, the vertical reactions at A and B are 575 lb each. The difference in readings at A is caused by a couple acting on the propeller when the engine is running. This couple tends to overturn the plane counterclockwise, which is opposite to the propeller's clockwise rotation. Determine the magnitude of this couple and the magnitude of the reaction force exerted at B when the engine is running.



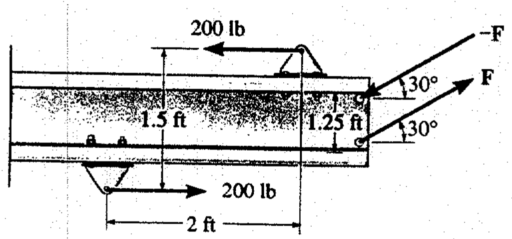
When the engine of the plane is turned on, the resulting couple moment exerts an additional force of $F = 650 - 575 = 75.0$ lb on wheel A and a lesser the reactive force on wheel B of $F = 75.0$ lb as well. Hence,

$$M = 75.0(12) = 900 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans}$$

The reactive force at wheel B is

$$R_B = 575 - 75.0 = 500 \text{ lb} \qquad \textbf{Ans}$$

4-78. Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?

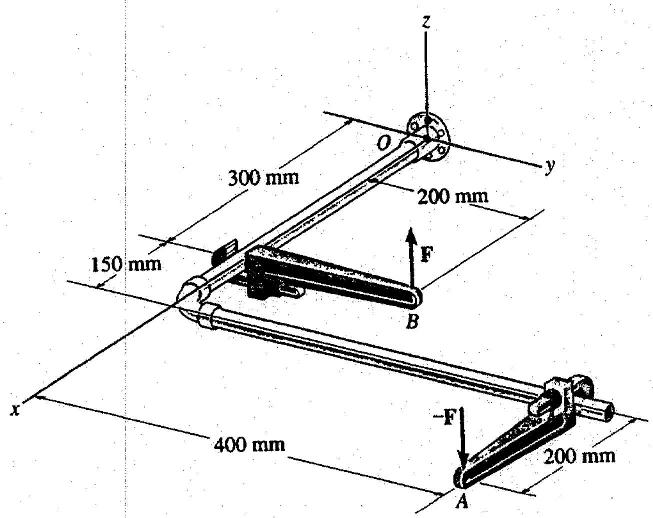


$\circlearrowleft + M_R = \Sigma M;\quad 450 = 200(1.5) + F\cos 30°(1.25)$

$$F = 139 \text{ lb} \qquad \textbf{Ans}$$

The resultant couple moment is a free vector. It can act at any point onthe beam.

4-79. Express the moment of the couple acting on the pipe assembly in Cartesian vector form. Solve the problem (a) using Eq. 4–13, and (b) summing the moment of each force about point O. Take F = {25k} N.



(a) $\mathbf{M}_C = \mathbf{r}_{AB} \times (25\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix}$$

$\mathbf{M}_C = \{-5\mathbf{i} + 8.75\mathbf{j}\}$ N·m **Ans**

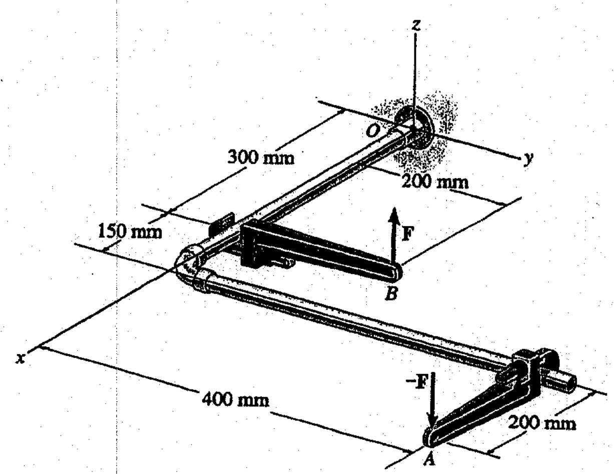
(b) $\mathbf{M}_C = \mathbf{r}_{OB} \times (25\mathbf{k}) + \mathbf{r}_{OA} \times (-25\mathbf{k})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.3 & 0.2 & 0 \\ 0 & 0 & 25 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.65 & 0.4 & 0 \\ 0 & 0 & -25 \end{vmatrix}$$

$\mathbf{M}_C = (5 - 10)\mathbf{i} + (-7.5 + 16.25)\mathbf{j}$

$\mathbf{M}_C = \{-5\mathbf{i} + 8.75\mathbf{j}\}$ N·m **Ans**

***4-80.** If the couple moment acting on the pipe has a magnitude of 400 N · m, determine the magnitude F of the vertical force applied to each wrench.



(a) $\mathbf{M}_C = \mathbf{r}_{AB} \times (F\mathbf{k})$

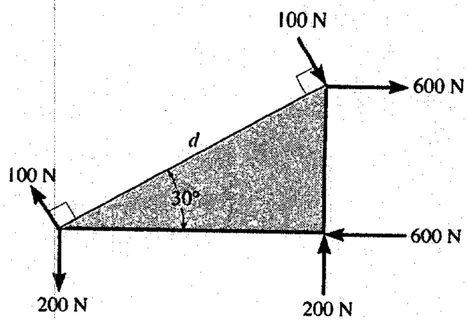
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.35 & -0.2 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$\mathbf{M}_C = \{-0.2F\mathbf{i} + 0.35F\mathbf{j}\}$ N·m

$M_C = \sqrt{(-0.2F)^2 + (0.35F)^2} = 400$

$$F = \frac{400}{\sqrt{(-0.2)^2 + (0.35)^2}} = 992 \text{ N}$$ **Ans**

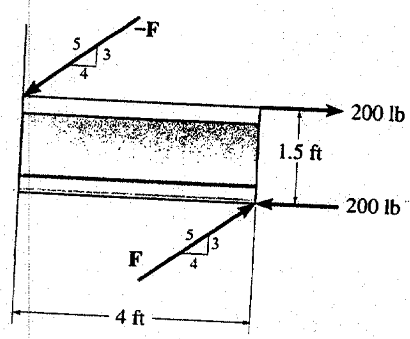
4-81. The ends of the triangular plate are subjected to three couples. Determine the plate dimension d so that the resultant couple is 350 N·m clockwise.



$\curvearrowleft + \; M_R = \Sigma M_A;\qquad -350 = 200(d\cos 30°) - 600(d\sin 30°) - 100d$

$d = 1.54$ m Ans

4-82. Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb·ft counterclockwise. Where on the beam does the resultant couple act?

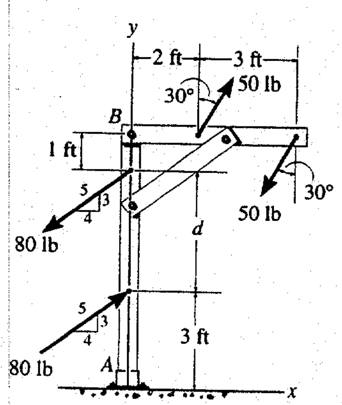


$\curvearrowleft + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$

$F = 167$ lb Ans

Resultant couple can act anywhere. Ans

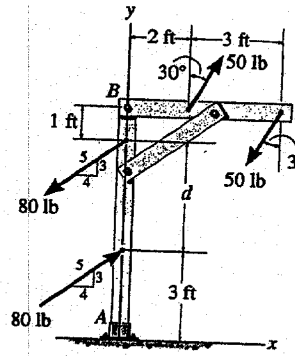
4-83. Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 80-lb couple forces.



$\curvearrowleft + M_C = -50\cos 30°(3) + \frac{4}{5}(80)(d) = 0$

$d = 2.03$ ft Ans

*4-84. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point A.



(a) $\mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$

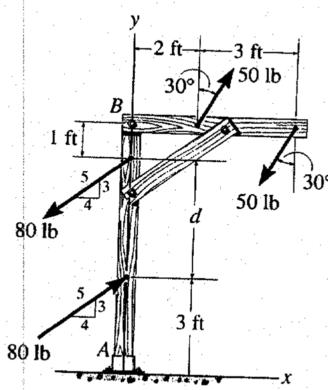
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50\sin 30^\circ & -50\cos 30^\circ & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ -\frac{4}{5}(80) & -\frac{3}{5}(80) & 0 \end{vmatrix}$$

$\mathbf{M}_C = \{126\mathbf{k}\}$ lb·ft **Ans**

(b) $\circlearrowleft + M_C = -\frac{4}{5}(80)(3) + \frac{4}{5}(80)(7) + 50\cos 30^\circ(2) - 50\cos 30^\circ(5)$

$M_C = 126$ lb·ft **Ans**

4-85. Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point B.



(a) $\mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$

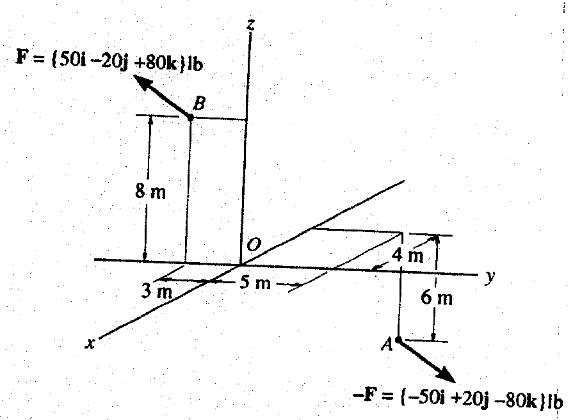
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50\sin 30^\circ & -50\cos 30^\circ & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 0 \\ \frac{4}{5}(80) & \frac{3}{5}(80) & 0 \end{vmatrix}$$

$\mathbf{M}_C = \{126\mathbf{k}\}$ lb·ft **Ans**

(b) $\circlearrowleft + M_C = 50\cos 30^\circ(2) - 50\cos 30^\circ(5) - \frac{4}{5}(80)(1) + \frac{4}{5}(80)(5)$

$M_C = 126$ lb·ft **Ans**

4-86. Determine the couple moment. Express the result as a Cartesian vector.



Position Vector :

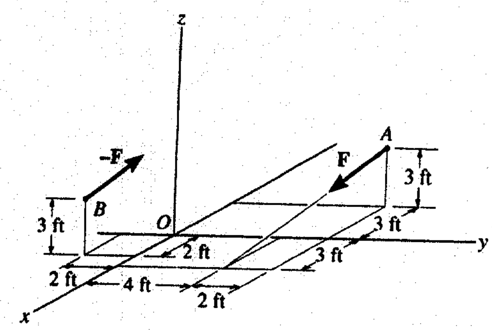
$$\mathbf{r}_{AB} = \{[0-(-4)]\mathbf{i} + (-3-5)\mathbf{j} + [8-(-6)]\mathbf{k}\} \text{ ft}$$
$$= \{4\mathbf{i} - 8\mathbf{j} + 14\mathbf{k}\} \text{ ft}$$

Couple Moment : With $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} + 80\mathbf{k}\}$ lb, applying Eq. 4 – 15, we have

$$\mathbf{M}_C = \mathbf{r}_{AB} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -8 & 14 \\ 50 & -20 & 80 \end{vmatrix}$$

$= \{-360\mathbf{i} + 380\mathbf{j} + 320\mathbf{k}\}$ lb · ft **Ans**

4-87. Determine the couple moment. Express the result as a Cartesian vector. Each force has a magnitude of $F = 120$ lb.



Position Vector and Force Vector:

$$\mathbf{r}_{BA} = \{(-3-2)\mathbf{i} + [6-(-2)]\mathbf{j} + (3-3)\mathbf{k}\}\ \text{ft} = \{-5\mathbf{i} + 8\mathbf{j}\}\ \text{ft}$$

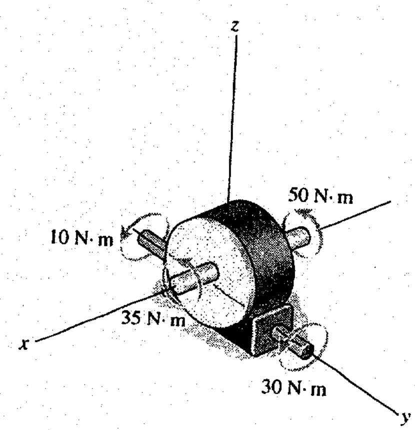
$$\mathbf{F} = 120\left(\frac{[3-(-3)]\mathbf{i} + (4-6)\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{[3-(-3)]^2 + (4-6)^2 + (0-3)^2}}\right) = \{102.86\mathbf{i} - 34.26\mathbf{j} - 51.43\mathbf{k}\}\ \text{lb}$$

Couple Moment: Applying Eq. 4 – 15, we have

$$\mathbf{M}_C = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -5 & 8 & 0 \\ 102.86 & -34.26 & -51.43 \end{vmatrix}$$

$$= \{-411\mathbf{i} - 257\mathbf{j} - 651\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$ **Ans**

***4-88.** The gear reducer is subjected to the four couple moments. Determine the magnitude of the resultant couple moment and its coordinate direction angles.



$$(M_R)_x = \Sigma M_x;\quad (M_R)_x = 35 + 50 = 85.0\ \text{N}\cdot\text{m}$$
$$(M_R)_y = \Sigma M_y;\quad (M_R)_y = 30 + 10 = 40.0\ \text{N}\cdot\text{m}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{(M_R)_x^2 + (M_R)_y^2} = \sqrt{85.0^2 + 40.0^2} = 93.941\ \text{N}\cdot\text{m} = 93.9\ \text{N}\cdot\text{m}$$ **Ans**

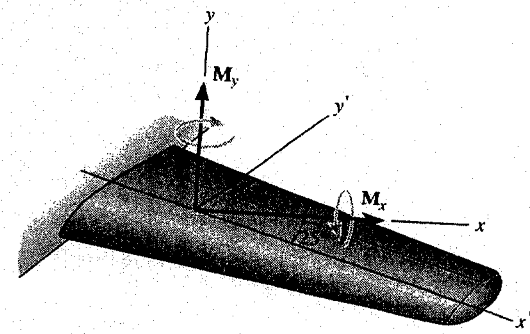
The coordinate direction angles are

$$\alpha = \cos^{-1}\left[\frac{(M_R)_x}{M_R}\right] = \cos^{-1}\left(\frac{85.0}{93.941}\right) = 25.2^\circ$$ **Ans**

$$\beta = \cos^{-1}\left[\frac{(M_R)_y}{M_R}\right] = \cos^{-1}\left(\frac{40.0}{93.941}\right) = 64.8^\circ$$ **Ans**

$$\gamma = \cos^{-1}\left[\frac{(M_R)_z}{M_R}\right] = \cos^{-1}\left(\frac{0}{93.941}\right) = 90.0^\circ$$ **Ans**

4-89. The main beam along the wing of an airplane is swept back at an angle of 25°. From load calculations it is determined that the beam is subjected to couple moments $M_x = 17$ kip·ft and $M_y = 25$ kip·ft. Determine the resultant couple moments created about the x' and y' axes. The axes all lie in the same horizontal plane.



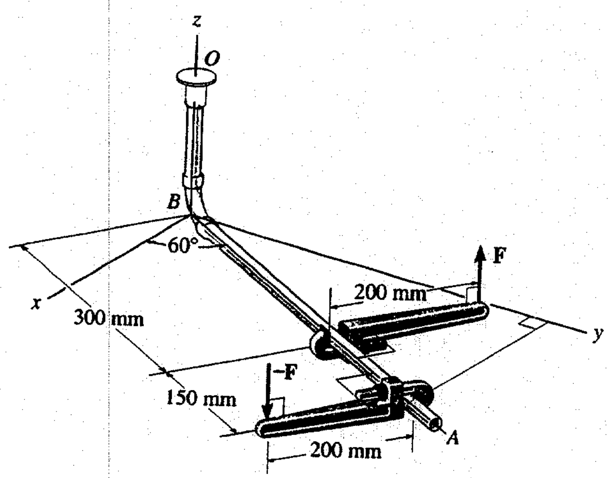
$(M_R)_{x'} = \Sigma M_{x'}$; $\quad (M_R)_{x'} = 17\cos 25° - 25\sin 25°$

$= 4.84$ kip · ft **Ans**

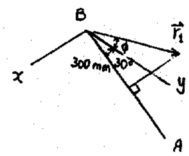
$(M_R)_{y'} = \Sigma M_{y'}$; $\quad (M_R)_{y'} = 17\sin 25° + 25\cos 25°$

$= 29.8$ kip · ft **Ans**

4-90. If $\mathbf{F} = \{100\mathbf{k}\}$ N, determine the couple moment that acts on the assembly. Express the result as a Cartesian vector. Member BA lies in the x-y plane.



$\phi = \tan^{-1}(\frac{2}{3}) - 30° = 3.69°$



$\mathbf{r}_1 = \{-360.6\sin 3.69°\mathbf{i} + 360.6\cos 3.69°\mathbf{j}\}$

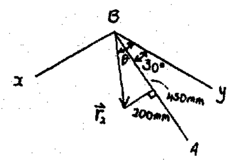
$= \{-23.21\mathbf{i} + 359.8\mathbf{j}\}$ mm

$\theta = \tan^{-1}(\frac{2}{4.5}) + 30° = 53.96°$

$\mathbf{r}_2 = \{492.4\sin 53.96°\mathbf{i} + 492.4\cos 53.96°\mathbf{j}\}$

$= \{398.2\mathbf{i} + 289.7\mathbf{j}\}$ mm

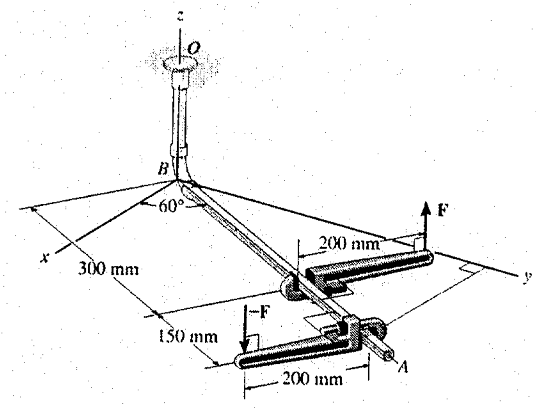
$\mathbf{M}_C = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$



$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & 100 \end{vmatrix}$$

$\mathbf{M}_C = \{7.01\mathbf{i} + 42.1\mathbf{j}\}$ N·m **Ans**

4-91. If the magnitude of the resultant couple moment is 15 N · m determine the magnitude F of the forces applied to the wrenches.



$$\phi = \tan^{-1}\left(\frac{2}{3}\right) - 30° = 3.69°$$

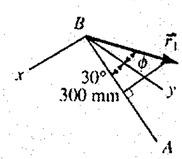
$$\mathbf{r}_1 = \{-360.6 \sin 3.69°\mathbf{i} + 360.6 \cos 3.69°\mathbf{j}\}$$

$$= \{-23.21\mathbf{i} + 359.8\mathbf{j}\} \text{ mm}$$

$$\theta = \tan^{-1}\left(\frac{2}{4.5}\right) + 30° = 53.96°$$

$$\mathbf{r}_2 = \{492.4 \sin 53.96°\mathbf{i} + 492.4 \cos 53.96°\mathbf{j}\}$$

$$= \{398.2\mathbf{i} + 289.7\mathbf{j}\} \text{ mm}$$



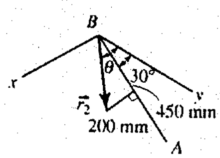
$$\mathbf{M}_C = (\mathbf{r}_1 - \mathbf{r}_2) \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -421.4 & 70.10 & 0 \\ 0 & 0 & F \end{vmatrix}$$

$$\mathbf{M}_C = \{0.07F\mathbf{i} + 0.421F\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$M_C = \sqrt{(0.07F)^2 + (0.421F)^2} = 15$$

$$F = \frac{15}{\sqrt{(0.07)^2 + (0.421)^2}} = 35.1 \text{ N} \qquad \textbf{Ans}$$



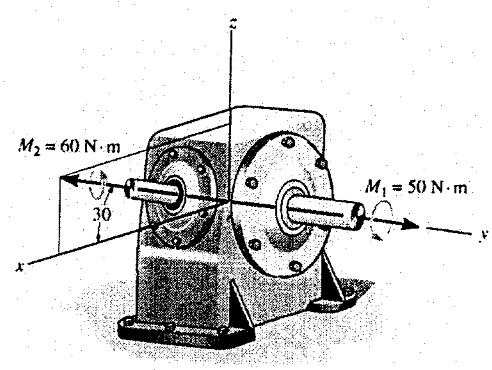
Also, align y' axis along BA.

$$\mathbf{M}_C = -F(0.15)\mathbf{i}' + F(0.4)\mathbf{j}'$$

$$15 = \sqrt{(F(0.15))^2 + (F(0.4))^2}$$

$$F = 35.1 \text{ N} \qquad \textbf{Ans}$$

4-92. The gear reducer is subjected to the couple moments shown. Determine the resultant couple moment and specify its magnitude and coordinate direction angles.



Express Each Couple Moment as a Cartesian Vector:

$$\mathbf{M}_1 = \{50\mathbf{j}\} \text{ N} \cdot \text{m}$$

$$\mathbf{M}_2 = 60(\cos 30°\mathbf{i} + \sin 30°\mathbf{k}) \text{ N} \cdot \text{m} = \{51.96\mathbf{i} + 30.0\mathbf{k}\} \text{ N} \cdot \text{m}$$

Resultant Couple Moment:

$$\mathbf{M}_R = \Sigma\mathbf{M}; \qquad \mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2$$

$$= \{51.96\mathbf{i} + 50.0\mathbf{j} + 30.0\mathbf{k}\} \text{ N} \cdot \text{m}$$

The magnitude of the resultant couple moment is

$$M_R = \sqrt{51.96^2 + 50.0^2 + 30.0^2}$$

$$= 78.102 \text{ N} \cdot \text{m} = 78.1 \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

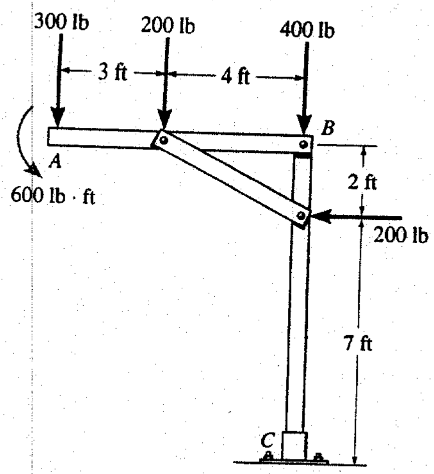
The coordinate direction angles are

$$\alpha = \cos^{-1}\left(\frac{51.96}{78.102}\right) = 48.3° \qquad \textbf{Ans}$$

$$\beta = \cos^{-1}\left(\frac{50.0}{78.102}\right) = 50.2° \qquad \textbf{Ans}$$

$$\gamma = \cos^{-1}\left(\frac{30.0}{78.102}\right) = 67.4° \qquad \textbf{Ans}$$

4-114. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = -200 \text{ lb} = 200 \text{ lb} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -300 - 200 - 400 = -900 \text{ lb} = 900 \text{ lb} \downarrow$

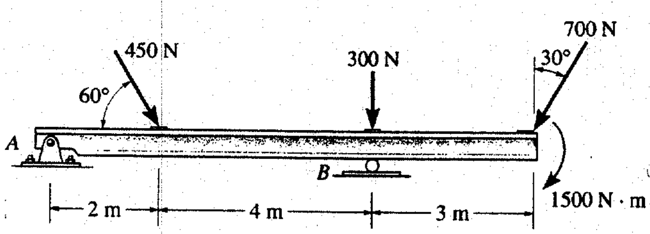
$F = \sqrt{(-200)^2 + (-900)^2} = 922 \text{ lb}$ **Ans**

$\theta = \tan^{-1}(\frac{900}{200}) = 77.5°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 900(x) = 200(3) + 400(7) + 200(2) - 600$

$x = \frac{3200}{900} = 3.56 \text{ ft}$ **Ans**

4-115. Replace the three forces acting on the beam by a single resultant force. Specify where the force acts, measured from end A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

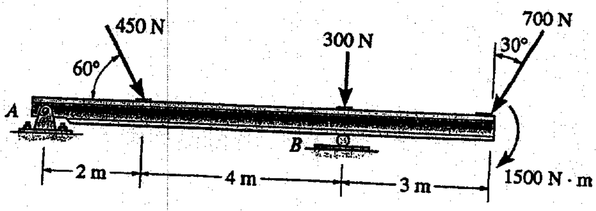
$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{1296}{125}) = 84.5°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 1296(x) = 450 \sin 60°(2) + 300(6) + 700 \cos 30°(9) + 1500$

$x = 7.36 \text{ m}$ **Ans**

***4-116.** Replace the three forces acting on the beam by a single resultant force. Specify where the force acts, measured from B.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

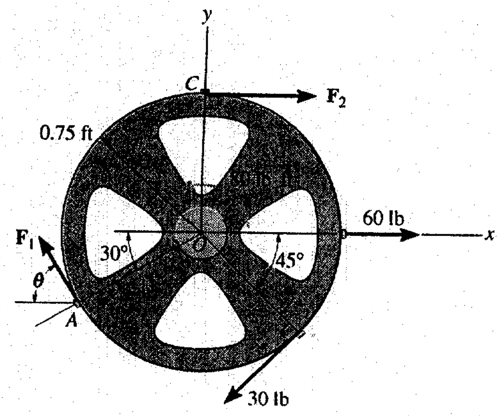
$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{1296}{125}) = 84.5°$ **Ans**

$\circlearrowleft + M_{RB} = \Sigma M_B;\quad 1296(x) = -450 \sin 60°(4) + 700 \cos 30°(3) + 1500$

$x = 1.36 \text{ m}$ (to the right) **Ans**

4-117. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ and the direction of $\mathbf{F}_1$ so that the loading creates a zero resultant force and couple moment on the wheel.



Force Summation :

$\xrightarrow{+} 0 = \Sigma F_x;\quad 0 = F_2 + 60 - F_1\cos\theta - 30\cos 45°$

$F_2 - F_1\cos\theta = -38.79 \quad [1]$

$+\uparrow 0 = \Sigma F_y;\quad 0 = F_1\sin\theta - 30\sin 45°$

$F_1\sin\theta = 21.21 \quad [2]$

Moment Summation :

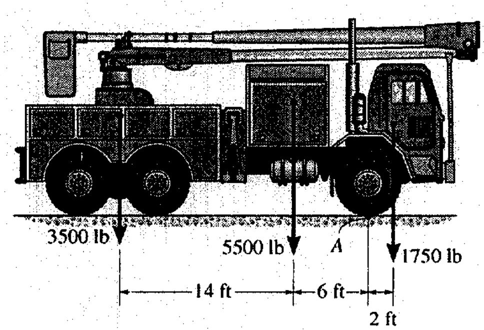
$\circlearrowleft + 0 = \Sigma M_O;\quad 0 = 80 - F_2(0.75) - 30(0.75) - F_1\sin\theta(0.75\cos 30°) - F_1\cos\theta(0.75\sin 30°)$

$0.6495F_1\sin\theta + 0.375F_1\cos\theta + 0.75F_2 = 57.5 \quad [3]$

Solving Eqs. [1], [2] and [3] yields

$F_2 = 25.9\text{ lb}\quad \theta = 18.1°\quad F_1 = 68.1\text{ lb}$ **Ans**

4-118. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and couple moment acting at point A.



$+\uparrow F_R = \Sigma F_y;\quad F_R = -1750 - 5500 - 3500$

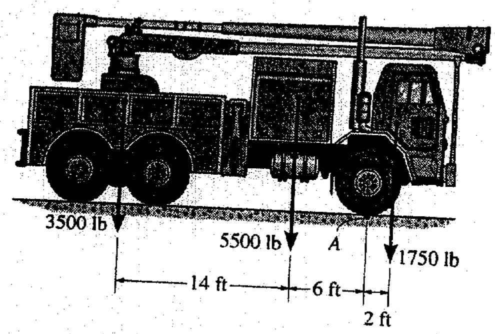
$= -10750\text{ lb} = 10.75\text{ kip}\downarrow$ **Ans**

$\circlearrowleft + M_{R_A} = \Sigma M_A;\quad M_{R_A} = 3500(20) + 5500(6) - 1750(2)$

$= 99500\text{ lb}\cdot\text{ft}$

$= 99.5\text{ kip}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

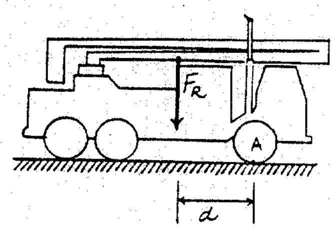
4-119. The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.



Equivalent Force :

$+\uparrow F_R = \Sigma F_y;\quad F_R = -1750 - 5500 - 3500$

$= -10750 \text{ lb} = 10.75 \text{ kip} \downarrow$ **Ans**



Location of Resultant Force From Point A :

$\curvearrowleft + M_{R_A} = \Sigma M_A;\quad 10750(d) = 3500(20) + 5500(6) - 1750(2)$

$d = 9.26 \text{ ft}$ **Ans**

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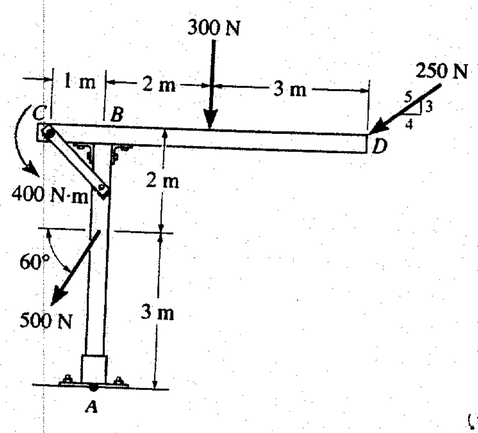
*4-120.** Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member AB, measured from A.



$\overset{+}{\rightarrow} \Sigma F_x = F_{Rx};\quad F_{Rx} = -250(\frac{4}{5}) - 500(\cos 60°) = -450 \text{ N} = 450 \text{ N} \leftarrow$

$+\uparrow \Sigma F_y = \Sigma F_y;\quad F_{Ry} = -300 - 250(\frac{3}{5}) - 500 \sin 60° = -883.0127 \text{ N} = 883.0127 \text{ N} \downarrow$

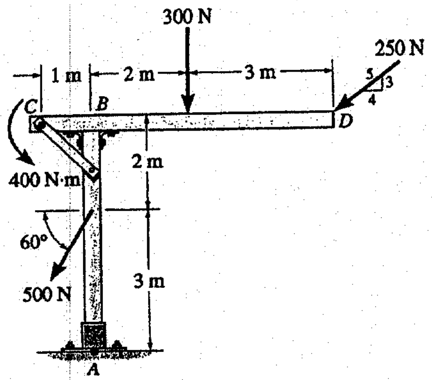
$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{883.0127}{450}) = 63.0°$

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad 450y = 400 + (500\cos 60°)(3) + 250(\frac{4}{5})(5) - 300(2) - 250(\frac{3}{5})(5)$

$y = \frac{800}{450} = 1.78 \text{ m}$ **Ans**

4-121. Replace the loading on the frame by a single resultant force. Specify where its line of action intersects member *CD*, measured from end *C*.



$\xrightarrow{+} \Sigma F_x = F_{Rx};\quad F_{Rx} = -250(\frac{4}{5}) - 500(\cos 60°) = -450 \text{ N} = 450 \text{ N} \leftarrow$

$+\uparrow \Sigma F_y = \Sigma F_y;\quad F_{Ry} = -300 - 250(\frac{3}{5}) - 500\sin 60° = -883.0127 \text{ N} = 883.0127 \text{ N} \downarrow$

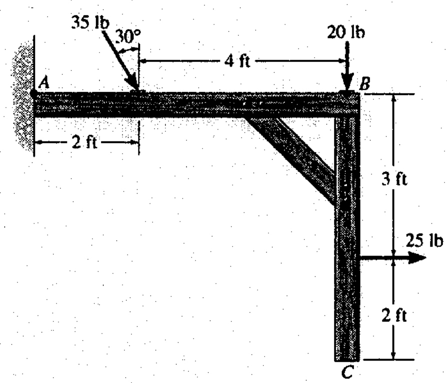
$F_R = \sqrt{(-450)^2 + (-883.0127)^2} = 991 \text{ N}$ **Ans**

$\theta = \tan^{-1}(\frac{883.0127}{450}) = 63.0°$

$\circlearrowleft + M_{RA} = \Sigma M_C;\quad 883.0127x = -400 + 300(3) + 250(\frac{3}{5})(6) + 500\cos 60°(2) + (500\sin 60°)(1)$

$x = \frac{2333}{883.0127} = 2.64 \text{ m}$ **Ans**

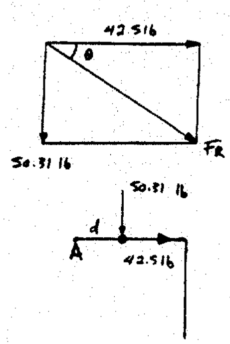
4-122. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member *AB*, measured from point *A*.



$\rightarrow F_{Rx} = \Sigma F_x;\quad F_{Rx} = 35\sin 30° + 25 = 42.5 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 35\cos 30° + 20 = 50.31 \text{ lb}$

$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

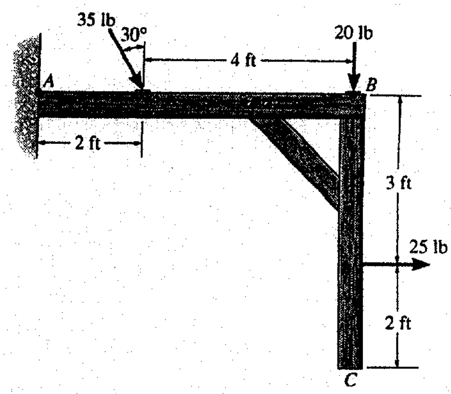


$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\quad 50.31(d) = 35\cos 30°(2) + 20(6) - 25(3)$

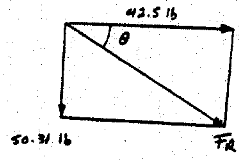
$d = 2.10 \text{ ft}$ **Ans**

4-123. Replace the force system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from point B.



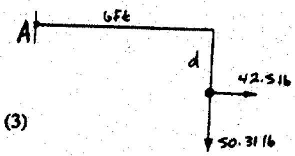
$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$



$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

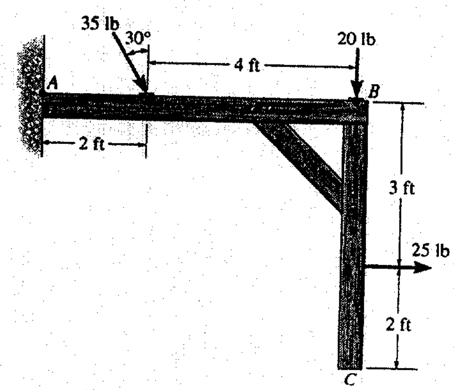
$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**



$\curvearrowleft + M_{RA} = \Sigma M_A;\quad 50.31(6) - 42.5(d) = 35 \cos 30° (2) + 20(6) - 25(3)$

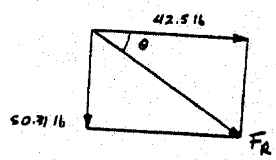
$d = 4.62 \text{ ft}$ **Ans**

***4-124.** Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.



$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 35 \sin 30° + 25 = 42.5 \text{ lb}$

$+\downarrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = 35 \cos 30° + 20 = 50.31 \text{ lb}$



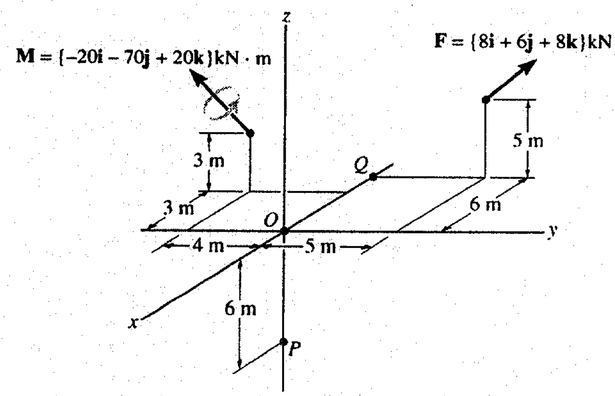
$F_R = \sqrt{(42.5)^2 + (50.31)^2} = 65.9 \text{ lb}$ **Ans**

$\theta = \tan^{-1}\left(\frac{50.31}{42.5}\right) = 49.8°$ **Ans**

$\curvearrowleft + M_{RA} = \Sigma M_A;\quad M_{RA} = 35\cos 30°(2) + 20(6) - 25(3)$

$M_{RA} = 106 \text{ lb}\cdot\text{ft}$ **Ans**

4-125. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point O. Express the results in Cartesian vector form.

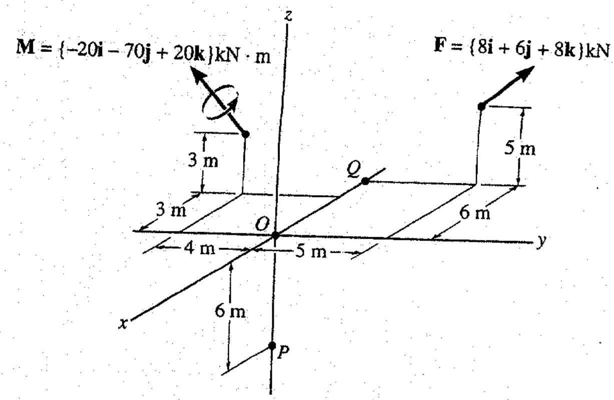


$\mathbf{F}_R = \Sigma \mathbf{F};\quad \mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\} \text{ kN}$ **Ans**

$$\mathbf{M}_{RO} = \Sigma \mathbf{M}_O;\quad \mathbf{M}_{RO} = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 5 & 5 \\ 8 & 6 & 8 \end{vmatrix}$$

$= \{-10\mathbf{i} + 18\mathbf{j} - 56\mathbf{k}\} \text{ kN}\cdot\text{m}$ **Ans**

4-126. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point P. Express the results in Cartesian vector form.

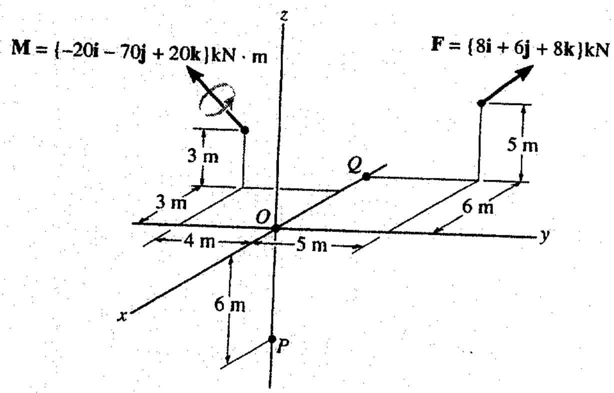


$\mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\}$ kN **Ans**

$$\mathbf{M}_{RP} = \Sigma\mathbf{M}_P = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -6 & 5 & 11 \\ 8 & 6 & 8 \end{vmatrix}$$

$$= \{-46\mathbf{i} + 66\mathbf{j} - 56\mathbf{k}\}\ \text{kN}\cdot\text{m}$$ **Ans**

4-127. Replace the force and couple-moment system by an equivalent resultant force and couple moment at point Q. Express the results in Cartesian vector form.

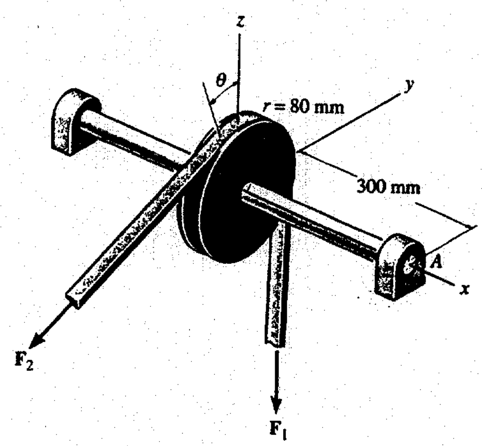


$\mathbf{F}_R = \{8\mathbf{i} + 6\mathbf{j} + 8\mathbf{k}\}$ kN **Ans**

$$\mathbf{M}_{RQ} = -20\mathbf{i} - 70\mathbf{j} + 20\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 5 & 5 \\ 8 & 6 & 8 \end{vmatrix}$$

$$= \{-10\mathbf{i} - 30\mathbf{j} - 20\mathbf{k}\}\ \text{kN}\cdot\text{m}$$ **Ans**

***4-128.** The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.



$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

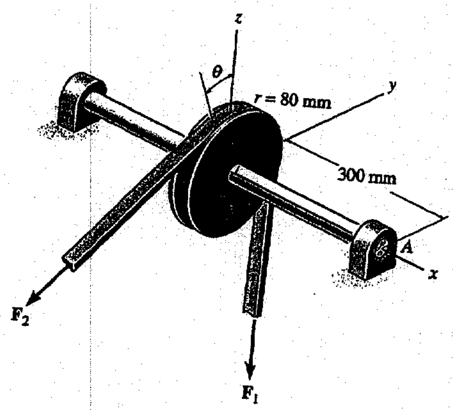
$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}$ N **Ans**

$\mathbf{M}_{RA} = \Sigma(\mathbf{r} \times \mathbf{F})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}$ N·m **Ans**

4-129. The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -40\cos 45°\mathbf{j} + (-40 - 40\sin 45°)\mathbf{k}$$

$$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}\ \text{N} \qquad \textbf{Ans}$$

$$\mathbf{r}_{AF1} = \{-0.3\mathbf{i} + 0.08\mathbf{j}\}\ \text{m}$$

$$\mathbf{r}_{AF2} = -0.3\mathbf{i} - 0.08\sin 45°\mathbf{j} + 0.08\cos 45°\mathbf{k}$$

$$= \{-0.3\mathbf{i} - 0.0566\mathbf{j} + 0.0566\mathbf{k}\}\ \text{m}$$

$$\mathbf{M}_{RA} = (\mathbf{r}_{AF1} \times \mathbf{F}_1) + (\mathbf{r}_{AF2} \times \mathbf{F}_2)$$

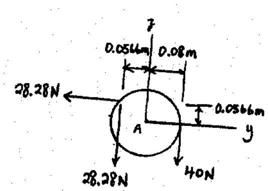
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40\cos 45° & -40\sin 45° \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

Also,

$$M_{RA_x} = \Sigma M_{A_x}$$
$$M_{RA_x} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)$$
$$M_{RA_x} = 0$$

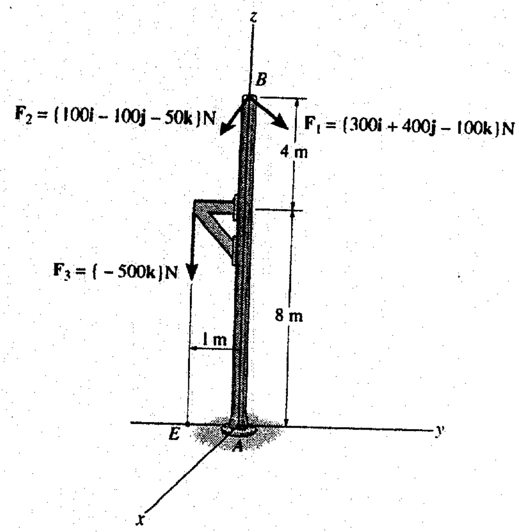
$$M_{RA_y} = \Sigma M_{A_y}$$
$$M_{RA_y} = -28.28(0.3) - 40(0.3)$$
$$M_{RA_y} = -20.5\ \text{N}\cdot\text{m}$$



$$M_{RA_z} = \Sigma M_{A_z}$$
$$M_{RA_z} = 28.28(0.3)$$
$$M_{RA_z} = 8.49\ \text{N}\cdot\text{m}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4-130. Replace the force system by an equivalent force and couple moment at point A.



$\mathbf{F}_R = \Sigma \mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= (300+100)\mathbf{i} + (400-100)\mathbf{j} + (-100-50-500)\mathbf{k}$

$= \{400\mathbf{i} + 300\mathbf{j} - 650\mathbf{k}\}$ N **Ans**

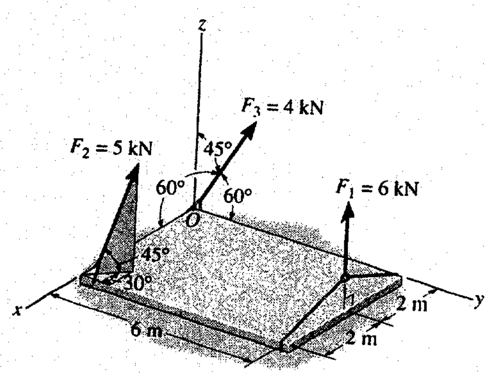
The position vectors are $\mathbf{r}_{AB} = \{12\mathbf{k}\}$ m and $\mathbf{r}_{AE} = \{-1\mathbf{j}\}$ m.

$\mathbf{M}_{R_A} = \Sigma \mathbf{M}_A; \quad \mathbf{M}_{R_A} = \mathbf{r}_{AB} \times \mathbf{F}_1 + \mathbf{r}_{AB} \times \mathbf{F}_2 + \mathbf{r}_{AE} \times \mathbf{F}_3$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 300 & 400 & -100 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -50 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix}$$

$= \{-3100\mathbf{i} + 4800\mathbf{j}\}$ N·m **Ans**

4-131. The slab is to be hoisted using the three slings shown. Replace the system of forces acting on slings by an equivalent force and couple moment at point O. The force $\mathbf{F}_1$, is vertical.



Force Vectors :

$\mathbf{F}_1 = \{6.00\mathbf{k}\}$ kN

$\mathbf{F}_2 = 5(-\cos 45°\sin 30°\mathbf{i} + \cos 45°\cos 30°\mathbf{j} + \sin 45°\mathbf{k})$

$= \{-1.768\mathbf{i} + 3.062\mathbf{j} + 3.536\mathbf{k}\}$ kN

$\mathbf{F}_3 = 4(\cos 60°\mathbf{i} + \cos 60°\mathbf{j} + \cos 45°\mathbf{k})$

$= \{2.00\mathbf{i} + 2.00\mathbf{j} + 2.828\mathbf{k}\}$ kN

Equivalent Force and Couple Moment At Point O :

$\mathbf{F}_R = \Sigma \mathbf{F}; \quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$= (-1.768 + 2.00)\mathbf{i} + (3.062 + 2.00)\mathbf{j}$

$+ (6.00 + 3.536 + 2.828)\mathbf{k}$

$= \{0.232\mathbf{i} + 5.06\mathbf{j} + 12.4\mathbf{k}\}$ kN **Ans**

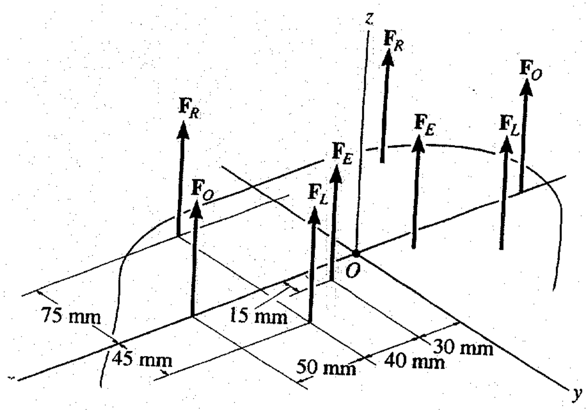
The position vectors are $\mathbf{r}_1 = \{2\mathbf{i} + 6\mathbf{j}\}$ m and $\mathbf{r}_2 = \{4\mathbf{i}\}$ m.

$\mathbf{M}_{R_O} = \Sigma \mathbf{M}_O; \quad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 6 & 0 \\ 0 & 0 & 6.00 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ -1.768 & 3.062 & 3.536 \end{vmatrix}$$

$= \{36.0\mathbf{i} - 26.1\mathbf{j} + 12.2\mathbf{k}\}$ kN·m **Ans**

***4-132.** A biomechanical model of the lumber region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y-z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.

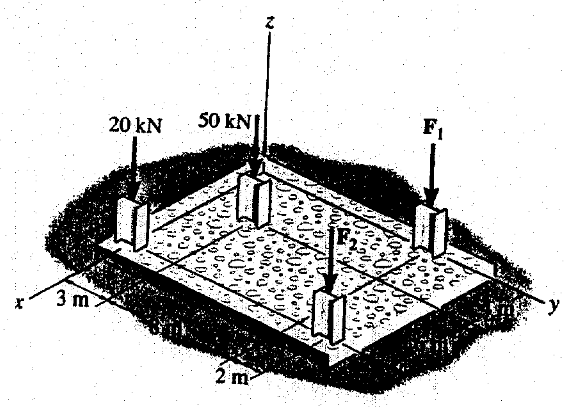


$\mathbf{F}_R = \Sigma \mathbf{F}_z;$ $\quad \mathbf{F}_R = \{2(35 + 45 + 23 + 32)\mathbf{k}\} = \{270\mathbf{k}\}$ N **Ans**

$\mathbf{M}_{RO_x} = \Sigma \mathbf{M}_{O_x};$ $\quad \mathbf{M}_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]\mathbf{i}$

$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}$ N·m **Ans**

4-133. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 30$ kN, $F_2 = 40$ kN.



$+\uparrow F_R = \Sigma F_z;$ $\quad F_R = -30 - 50 - 40 - 20 = -140$ kN $= 140$ kN $\downarrow$ **Ans**

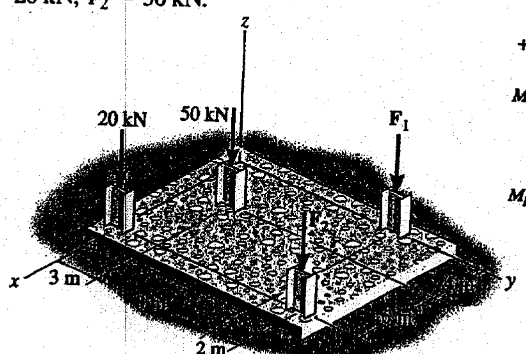
$(M_R)_x = \Sigma M_x;$ $\quad -140y = -50(3) - 30(11) - 40(13)$

$y = 7.14$ m **Ans**

$(M_R)_y = \Sigma M_y;$ $\quad 140x = 50(4) + 20(10) + 40(10)$

$x = 5.71$ m **Ans**

4-134. The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 20$ kN, $F_2 = 50$ kN.



$+\downarrow F_R = \Sigma F_z;$ $\quad F_R = 20 + 50 + 20 + 50 = 140$ kN **Ans**

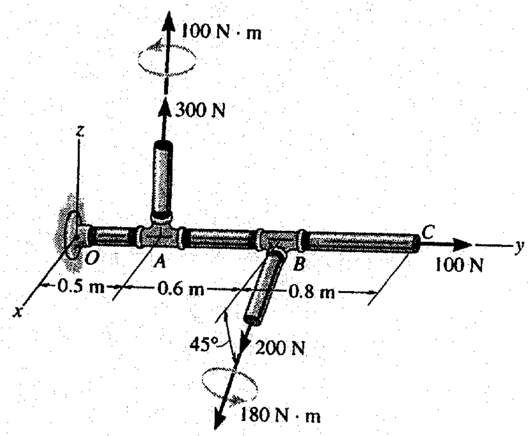
$M_{Roy} = \Sigma M_y;$ $\quad 140(x) = (50)(4) + 20(10) + 50(10)$

$x = 6.43$ m **Ans**

$M_{Rox} = \Sigma M_x;$ $\quad -140(y) = -(50)(3) - 20(11) - 50(13)$

$y = 7.29$ m **Ans**

***4-135.** Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



Force And Moment Vectors :

$\mathbf{F}_1 = \{300\mathbf{k}\}$ N $\qquad \mathbf{F}_3 = \{100\mathbf{j}\}$ N

$$\mathbf{F}_2 = 200\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\} \text{ N} = \{141.42\mathbf{i} - 141.42\mathbf{k}\} \text{ N}$$

$\mathbf{M}_1 = \{100\mathbf{k}\}$ N · m

$$\mathbf{M}_2 = 180\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\} \text{ N}\cdot\text{m} = \{127.28\mathbf{i} - 127.28\mathbf{k}\} \text{ N}\cdot\text{m}$$

Equivalent Force and Couple Moment At Point O :

$\mathbf{F}_R = \Sigma\mathbf{F};\quad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$= 141.42\mathbf{i} + 100.0\mathbf{j} + (300 - 141.42)\mathbf{k}$$

$$= \{141\mathbf{i} + 100\mathbf{j} + 159\mathbf{k}\} \text{ N}$$ **Ans**

The position vectors are $\mathbf{r}_1 = \{0.5\mathbf{j}\}$ m and $\mathbf{r}_2 = \{1.1\mathbf{j}\}$ m.

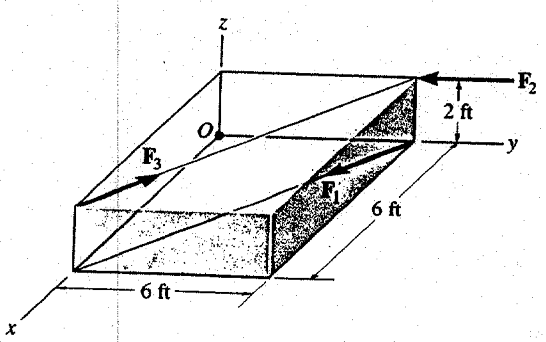
$\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O;\quad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{M}_1 + \mathbf{M}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.42 & 0 & -141.42 \end{vmatrix}$$

$$+ 100\mathbf{k} + 127.28\mathbf{i} - 127.28\mathbf{k}$$

$$= \{122\mathbf{i} - 183\mathbf{k}\} \text{ N}\cdot\text{m}$$ **Ans**

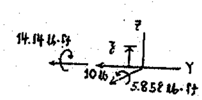
***4-136.** The three forces acting on the block each have a magnitude of 10 lb. Replace this system by a wrench and specify the point where the wrench intersects the z axis, measured from point O.



$\mathbf{F}_R = \{-10\mathbf{j}\}$ lb

$\mathbf{M}_O = (6\mathbf{j} + 2\mathbf{k}) \times (-10\mathbf{j}) + 2(10)(-0.707\mathbf{i} - 0.707\mathbf{j})$

$= \{5.858\mathbf{i} - 14.14\mathbf{j}\}$ lb·ft



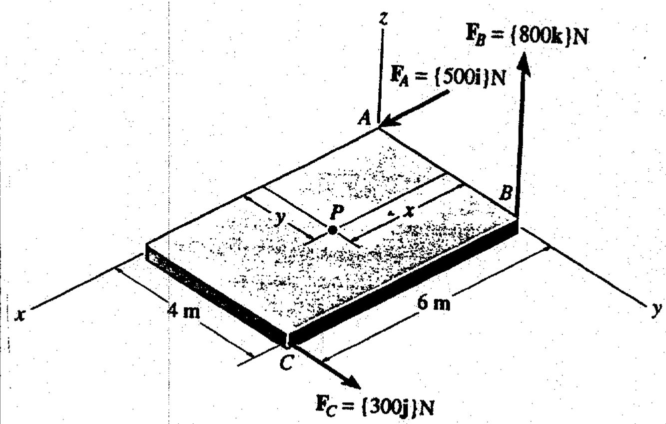
Require

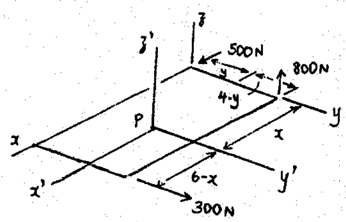
$z = \dfrac{5.858}{10} = 0.586$ ft **Ans**

$\mathbf{F}_w = \{-10\mathbf{j}\}$ lb **Ans**

$\mathbf{M}_w = \{-14.1\mathbf{j}\}$ lb·ft **Ans**

4-137. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where its line of action intersects the plate.





$\mathbf{F}_R = \{500\mathbf{i} + 300\mathbf{j} + 800\mathbf{k}\}$ N

$F_R = \sqrt{(500)^2 + (300)^2 + (800)^2} = 990$ N **Ans**

$\mathbf{u}_{FR} = \{0.5051\mathbf{i} + 0.3030\mathbf{j} + 0.8081\mathbf{k}\}$

$M_{R_{x'}} = \Sigma M_{x'}; \quad M_{R_{x'}} = 800(4-y)$

$M_{R_{y'}} = \Sigma M_{y'}; \quad M_{R_{y'}} = 800x$

$M_{R_{z'}} = \Sigma M_{z'}; \quad M_{R_{z'}} = 500y + 300(6-x)$

Since $\mathbf{M}_R$ also acts in the direction of $\mathbf{u}_{FR}$,

$M_R(0.5051) = 800(4-y)$

$M_R(0.3030) = 800x$

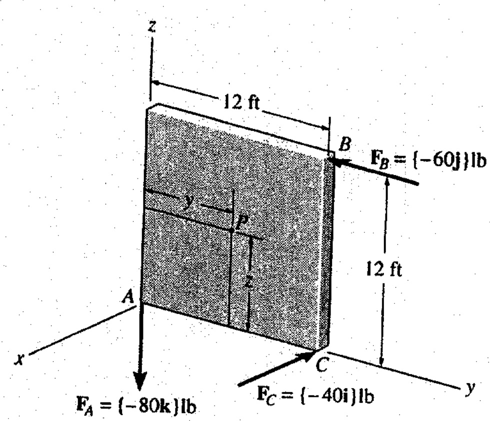
$M_R(0.8081) = 500y + 300(6-x)$

$M_R = 3.07$ kN·m **Ans**

$x = 1.16$ m **Ans**

$y = 2.06$ m **Ans**

4-138. Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(y, z)$ where its line of action intersects the plate.



Resultant Force Vector :

$\mathbf{F}_R = \{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}\}$ lb

$F_R = \sqrt{(-40)^2 + (-60)^2 + (-80)^2} = 107.70 \text{ lb} = 108$ lb **Ans**

$$\mathbf{u}_{F_R} = \frac{-40\mathbf{i} - 60\mathbf{j} - 80\mathbf{k}}{107.70} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$$

Resultant Moment: The line of action of $\mathbf{M}_R$ of the wrench is parallel to the line of action of $\mathbf{F}_R$. Assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Therefore, $\mathbf{u}_{M_R} = -0.3714\mathbf{i} - 0.5571\mathbf{j} - 0.7428\mathbf{k}$.

$(M_R)_{x'} = \Sigma M_{x'}; \quad -0.3714M_R = 60(12-z) + 80y$ [1]

$(M_R)_{y'} = \Sigma M_{y'}; \quad -0.5571M_R = 40z$ [2]

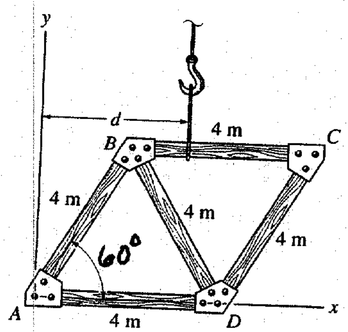
$(M_R)_{z'} = \Sigma M_{z'}; \quad -0.7428M_R = 40(12-y)$ [3]

Solving Eqs.[1], [2], and [3] yields :

$M_R = -624$ lb·ft $\quad z = 8.69$ ft $\quad y = 0.414$ ft **Ans**

The negative sign indicates that the line of action for $\mathbf{M}_R$ is directed in the opposite sense to that of $\mathbf{F}_R$.

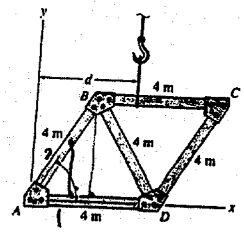
*9-48. The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



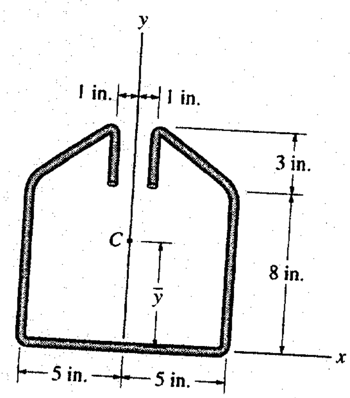
$\Sigma \tilde{x} M = 4(7)(1+4+2+3+5) = 420$ kg·m

$\Sigma M = 4(7)(5) = 140$ kg

$d = \bar{x} = \frac{\Sigma \tilde{x} M}{\Sigma M} = \frac{420}{140} = 3$ m **Ans**



9-49. Locate the centroid for the wire. Neglect the thickness of the material and slight bends at the corners.

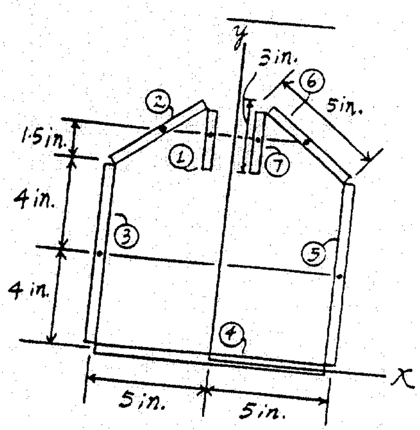


Centroid : The length of each segment and its respective centroid are tabulated below.

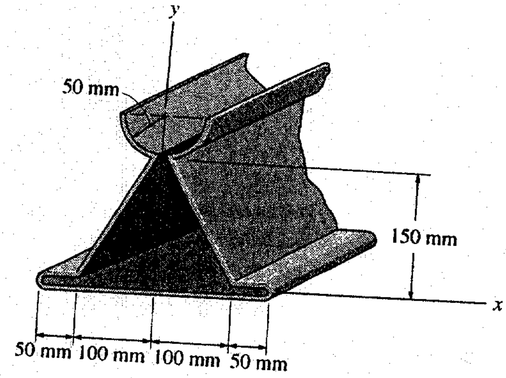
Segment	L(in.)	$\tilde{y}$(in.)	$\tilde{y}L$(in²)
1	3	9.5	28.5
2	5	9.5	47.5
3	8	4	32.0
4	10	0	0
5	8	4	32.0
6	5	9.5	47.5
7	3	9.5	28.5
Σ	42.0		216.0

Due to symmetry about y axis, $\bar{x} = 0$ **Ans**

$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{216.0}{42.0} = 5.143$ in. $= 5.14$ in. **Ans**



9-50. Locate the centroid ($\bar{x}$, $\bar{y}$) of the metal cross section. Neglect the thickness of the material and slight bends at the corners.

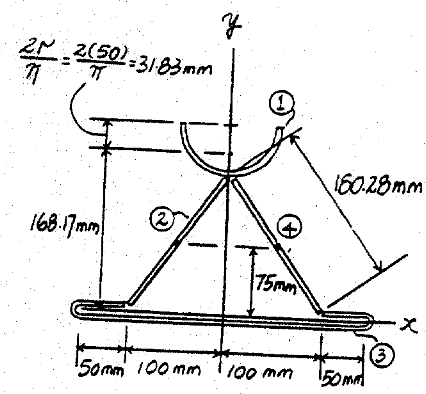


Centroid : The length of each segment and its respective centroid are tabulated below.

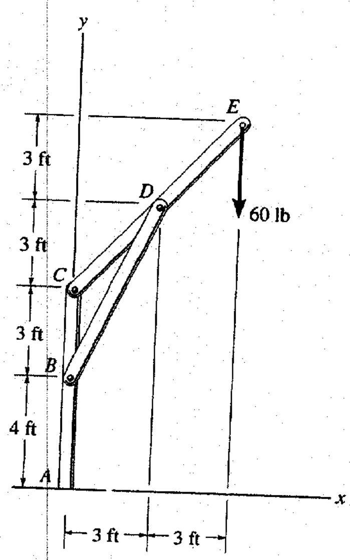
Segment	L(mm)	$\bar{y}$(mm)	$\bar{y}L$(mm^2)
1	50π	168.17	26415.93
2	180.28	75	13520.82
3	400	0	0
4	180.28	75	13520.82
Σ	917.63		53457.56

Due to symmetry about y axis, $\bar{x} = 0$ **Ans**

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{53457.56}{917.63} = 58.26 \text{ mm} = 58.3 \text{ mm}$$ **Ans**



9-51. The three members of the frame each have a weight per unit length of 4 lb/ft. Locate the position ($\bar{x}$, $\bar{y}$) of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the fixed support A.



$$\Sigma \bar{x} W = 1.5(4)\sqrt{45} + 3(4)\sqrt{72} = 142.073 \text{ lb}\cdot\text{ft}$$

$$\Sigma W = 4(7) + 4\sqrt{45} + 4\sqrt{72} = 88.774 \text{ lb}$$

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{142.073}{88.774} = 1.60 \text{ ft}$$ **Ans**

$$\Sigma \bar{y} W = 3.5(4)(7) + 7(4)\sqrt{45} + 10(4)\sqrt{72} = 625.241 \text{ lb.ft}$$

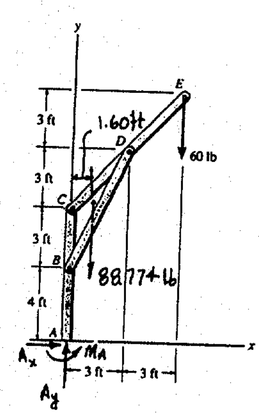
$$\bar{y} = \frac{\Sigma \bar{y} W}{\Sigma W} = \frac{625.241}{88.774} = 7.04 \text{ ft}$$ **Ans**

$\xrightarrow{+} \Sigma F_x = 0;$ $A_x = 0$ **Ans**

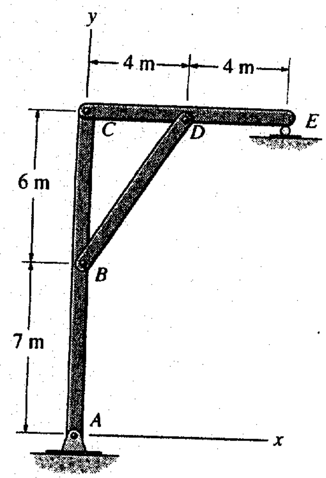
$+\uparrow \Sigma F_y = 0;$ $A_y = 88.774 + 60 = 149 \text{ lb}$ **Ans**

$\circlearrowleft + \Sigma M_A = 0;$ $-60(6) - 88.774(1.60) + M_A = 0$

$M_A = 502 \text{ lb}\cdot\text{ft}$ **Ans**

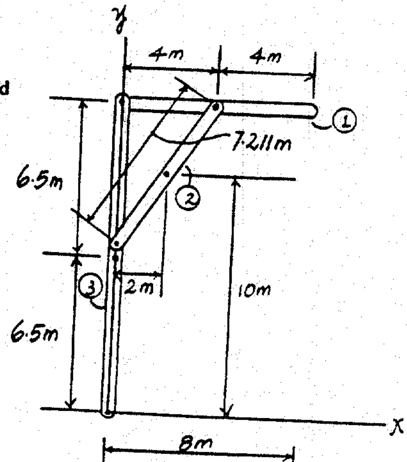


***9-52.** Each of the three members of the frame has a mass per unit length of 6 kg/m. Locate the position $(\bar{x}, \bar{y})$ of the center of gravity. Neglect the size of the pins at the joints and the thickness of the members. Also, calculate the reactions at the pin A and roller E.



Centroid : The length of each segment and its respective centroid are tabulated below.

Segment	L(m)	$\bar{x}$(m)	$\bar{y}$(m)	$\bar{x}L$(m²)	$\bar{y}L$(m²)
1	8	4	13	32.0	104.0
2	7.211	2	10	14.42	72.11
3	13	0	6.5	0	84.5
Σ	28.211			46.42	260.61



Thus,

$$\bar{x} = \frac{\Sigma \bar{x}L}{\Sigma L} = \frac{46.42}{28.211} = 1.646 \text{ m} = 1.65 \text{ m} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \bar{y}L}{\Sigma L} = \frac{260.61}{28.211} = 9.238 \text{ m} = 9.24 \text{ m} \qquad \textbf{Ans}$$

Equations of Equilibrium : The total weight of the frame is $W = 28.211(6)(9.81) = 1660.51$ N.

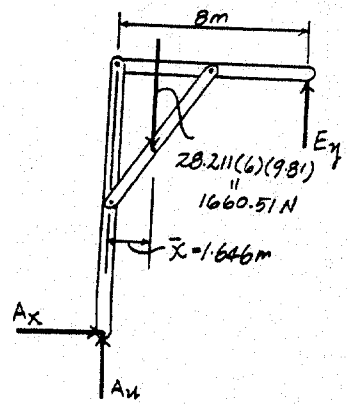
$+\Sigma M_A = 0;\quad E_y(8) - 1660.51(1.646) = 0$

$E_y = 341.55 \text{ N} = 342 \text{ N}$ **Ans**

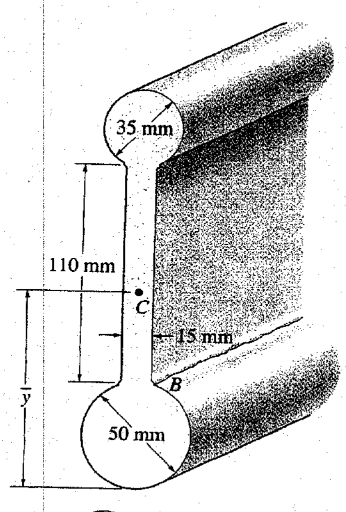
$+\uparrow \Sigma F_y = 0;\quad A_y + 341.55 - 1660.51 = 0$

$A_y = 1318.95 \text{ N} = 1.32 \text{ kN}$ **Ans**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x = 0$ **Ans**



9-53. Determine the location $\bar{y}$ of the centroid of the beam's cross-sectional area. Neglect the size of the corner [w]elds at A and B for the calculation.

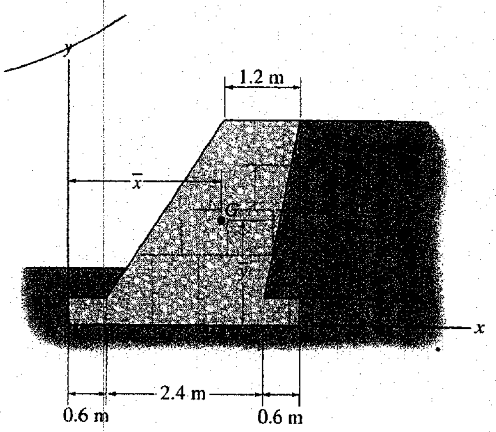


$$\Sigma \bar{y}A = \pi(25)^2(25) + 15(110)(50 + 55) + \pi\left(\frac{35}{2}\right)^2\left(50 + 110 + \frac{35}{2}\right) = 393\,112 \text{ mm}^3$$

$$\Sigma A = \pi(25)^2 + 15(110) + \pi\left(\frac{35}{2}\right)^2 = 4575.6 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{393\,112}{4575.6} = 85.9 \text{ mm} \qquad \textbf{Ans}$$

9-54. The gravity wall is made of concrete. Determine the location $(\bar{x}, \bar{y})$ of the center of gravity G for the wall.



$$\Sigma \tilde{x}A = 1.8(3.6)(0.4) + 2.1(3)(3) - 3.4\left(\frac{1}{2}\right)(3)(0.6) - 1.2\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 15.192\ \text{m}^3$$

$$\Sigma \tilde{y}A = 0.2(3.6)(0.4) + 1.9(3)(3) - 1.4\left(\frac{1}{2}\right)(3)(0.6) - 2.4\left(\frac{1}{2}\right)(1.8)(3)$$

$$= 9.648\ \text{m}^3$$

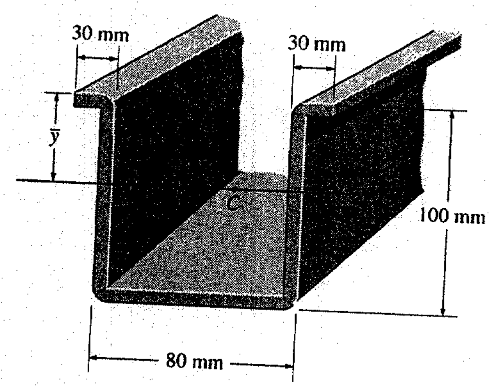
$$\Sigma A = 3.6(0.4) + 3(3) - \frac{1}{2}(3)(0.6) - \frac{1}{2}(1.8)(3)$$

$$= 6.84\ \text{m}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{15.192}{6.84} = 2.22\ \text{m} \quad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{9.648}{6.84} = 1.41\ \text{m} \quad \textbf{Ans}$$

9-55. An aluminum strut has a cross section referred to as a deep hat. Locate the centroid $\bar{y}$ of its area. Each segment has a thickness of 10 mm.

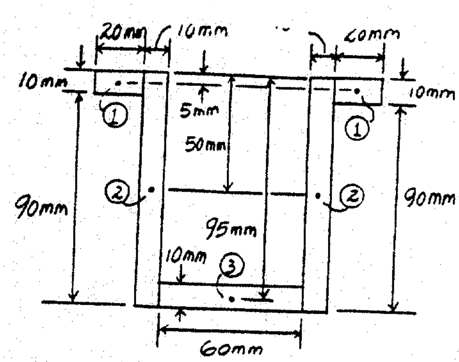


Centroid : The area of each segment and its respective centroid are tabulated below.

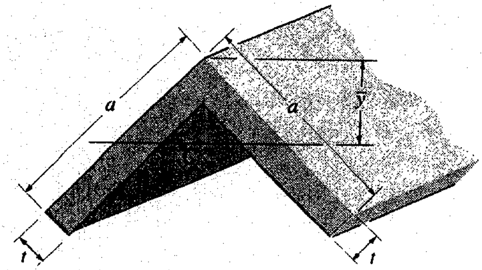
Segment	A (mm²)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm³)
1	40(10)	5	2 000
2	100(20)	50	100 000
3	60(10)	95	57 000
Σ	3 000		159 000

Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{159\,000}{3\,000} = 53.0\ \text{mm} \quad \textbf{Ans}$$



***9-56.** Locate the centroid $\bar{y}$ for the cross-sectional area of the angle.



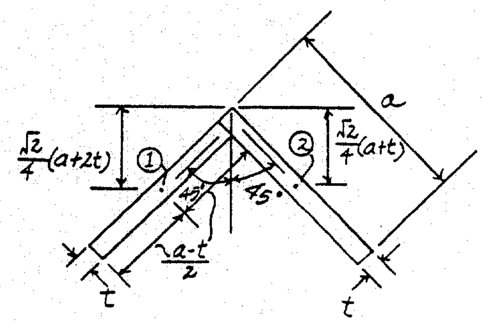
Centroid **:** The area and the centroid for segments 1 and 2 are

$$A_1 = t(a-t)$$

$$\tilde{y}_1 = \left(\frac{a-t}{2}+\frac{t}{2}\right)\cos 45° + \frac{t}{2\cos 45°} = \frac{\sqrt{2}}{4}(a+2t)$$

$$A_2 = at$$

$$\tilde{y}_2 = \left(\frac{a}{2}-\frac{t}{2}\right)\cos 45° + \frac{t}{2\cos 45°} = \frac{\sqrt{2}}{4}(a+t)$$



Listed in a tabular form, we have

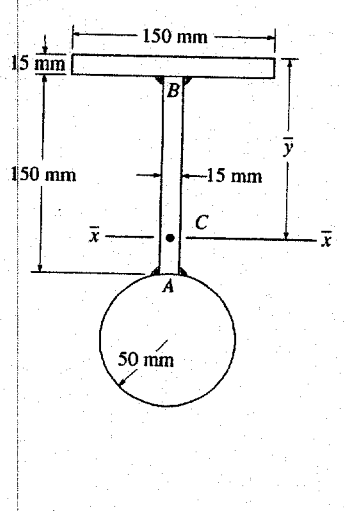
Segment	A	$\tilde{y}$	$\tilde{y}A$
1	$t(a-t)$	$\frac{\sqrt{2}}{4}(a+2t)$	$\frac{\sqrt{2}t}{4}(a^2+at-2t^2)$
2	at	$\frac{\sqrt{2}}{4}(a+t)$	$\frac{\sqrt{2}t}{4}(a^2+at)$
Σ	$t(2a-t)$		$\frac{\sqrt{2}t}{2}(a^2+at-t^2)$

Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{\frac{\sqrt{2}t}{2}(a^2+at-t^2)}{t(2a-t)}$$

$$= \frac{\sqrt{2}(a^2+at-t^2)}{2(2a-t)} \qquad \textbf{Ans}$$

9-57. Determine the location $\bar{y}$ of the centroidal axis $\bar{x}\,\bar{x}$ of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.



$$\Sigma\tilde{y}A = 7.5(15)(150) + 90(150)(15) + 215(\pi)(50)^2$$

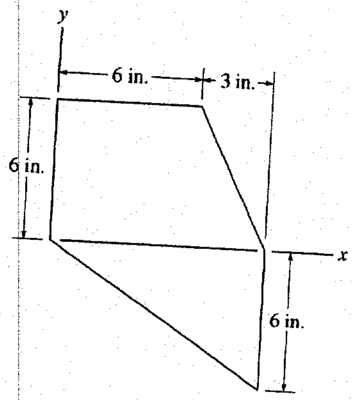
$$= 1\,907\,981.05\ \text{mm}^3$$

$$\Sigma A = 15(150) + 150(15) + \pi(50)^2$$

$$= 12\,353.98\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{1\,907\,981.05}{12\,353.98} = 154\ \text{mm} \qquad \textbf{Ans}$$

9-58. Determine the location $(\bar{x}, \bar{y})$ of the centroid C of the area.



$$\Sigma \tilde{x} A = 3(6)(6) + 7(\tfrac{1}{2})(6)(3) + 6(\tfrac{1}{2})(9)(6) = 333 \text{ in}^3$$

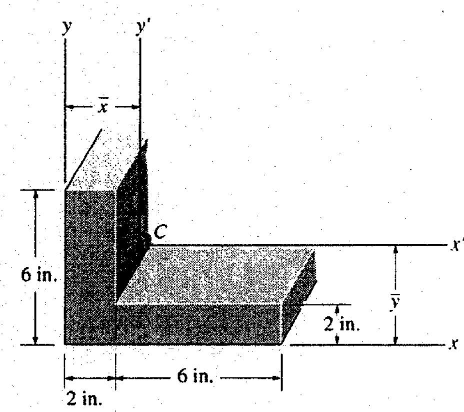
$$\Sigma \tilde{y} A = 3(6)(6) + 2(\tfrac{1}{2})(6)(3) - 2(\tfrac{1}{2})(9)(6) = 72 \text{ in}^3$$

$$\Sigma A = 6(6) + \tfrac{1}{2}(6)(3) + \tfrac{1}{2}(9)(6) = 72 \text{ in}^2$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{333}{72} = 4.625 = 4.62 \text{ in.}$$ **Ans**

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{72}{72} = 1.00 \text{ in.}$$ **Ans**

9-59. Locate the centroid $(\bar{x}, \bar{y})$ for the angle's cross-sectional area.



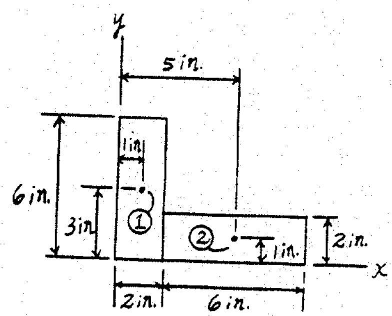
Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (in²)	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$ (in³)	$\tilde{y}A$ (in³)
1	6(2)	1	3	12.0	36.0
2	6(2)	5	1	60.0	12.0
Σ	24.0			72.0	48.0

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{72.0}{24.0} = 3.00 \text{ in.}$$ **Ans**

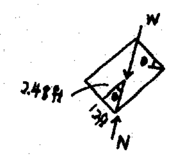
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{48.0}{24.0} = 2.00 \text{ in.}$$ **Ans**

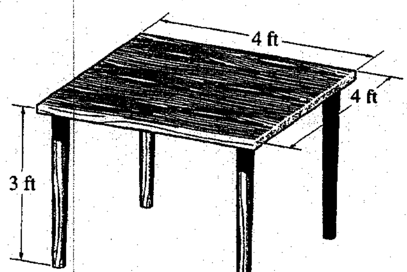


***9-60.** The wooden table is made from a square board having a weight of 15 lb. Each of the legs weighs 2 lb and is 3 ft long. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.

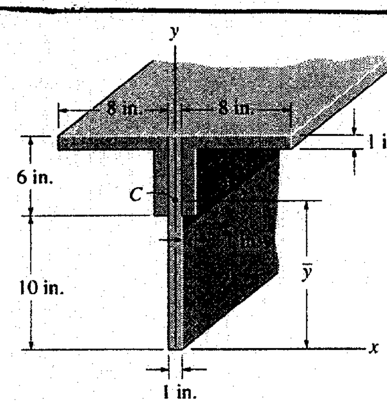
$$\bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} = \frac{15(3) + 4(2)(1.5)}{15 + 4(2)} = 2.48 \text{ ft} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left(\frac{2}{2.48}\right) = 38.9° \qquad \textbf{Ans}$$



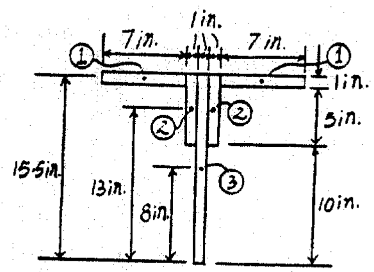


9-61. Locate the centroid $\bar{y}$ of the cross-sectional area of the beam.



Centroid : The area of each segment and its respective centroid are tabulated below.

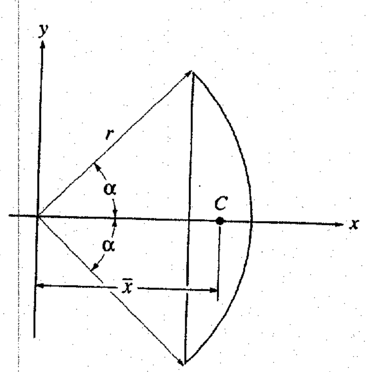
Segment	A (in^2)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in^3)
1	14(1)	15.5	217.0
2	6(2)	13	156.0
3	16(1)	8	128.0
Σ	42.0		501.0



Thus,

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{501.0}{42.0} = 11.93 \text{ in.} = 11.9 \text{ in.} \qquad \textbf{Ans}$$

9-62. Determine the location $\bar{x}$ of the centroid C of the shaded area which is part of a circle having a radius r.



$$\Sigma \tilde{x} A = \frac{1}{2} r^2 \alpha \left(\frac{2r}{3\alpha} \sin\alpha\right) - \frac{1}{2}(r \sin\alpha)(r\cos\alpha)\left(\frac{2}{3} r\cos\alpha\right)$$

$$= \frac{r^3}{3}\sin\alpha - \frac{r^3}{3}\sin\alpha\cos^2\alpha$$

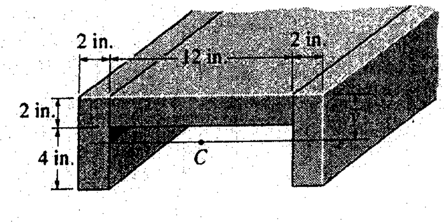
$$= \frac{r^3}{3}\sin^3\alpha$$

$$\Sigma A = \frac{1}{2}r^2\alpha - \frac{1}{2}(r\sin\alpha)(r\cos\alpha)$$

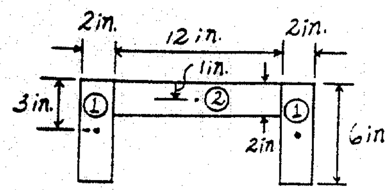
$$= \frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)$$

$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\frac{r^3}{3}\sin^3\alpha}{\frac{1}{2}r^2\left(\alpha - \frac{\sin 2\alpha}{2}\right)} = \frac{\frac{2}{3} r \sin^3\alpha}{\alpha - \frac{\sin 2\alpha}{2}} \qquad \textbf{Ans}$$

9-63. Locate the centroid $\bar{y}$ of the channel's cross-sectional area.



Centroid **:** The area of each segment and its respective centroid are tabulated below.

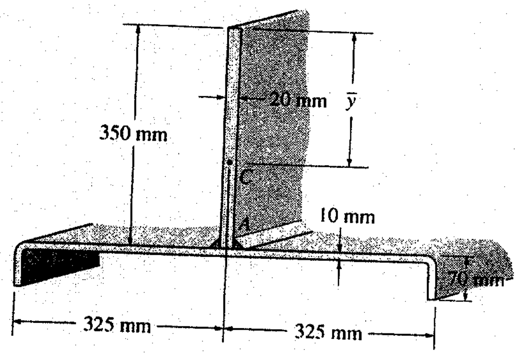


Segment	A (in²)	$\tilde{y}$ (in.)	$\tilde{y}A$ (in³)
1	6(4)	3	72.0
2	12(2)	1	24.0
Σ	48.0		96.0

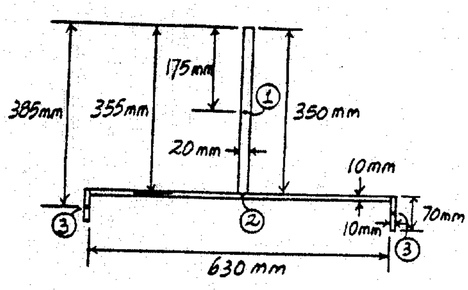
Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{96.0}{48.0} = 2.00 \text{ in.}$$ **Ans**

*9-64. Locate the centroid $\bar{y}$ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at A.



Centroid **:** The area of each segment and its respective centroid are tabulated below.

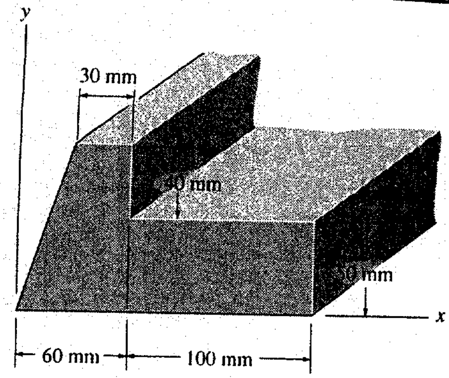


Segment	A (mm²)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm³)
1	350(20)	175	1 225 000
2	630(10)	355	2 236 500
3	70(20)	385	539 000
Σ	14 700		4 000 500

Thus,

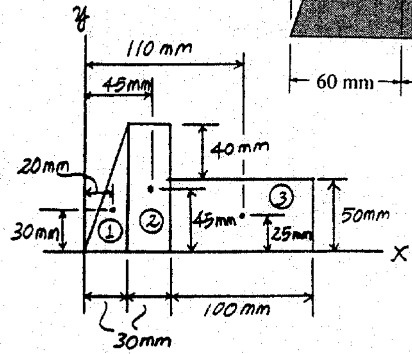
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{4\,000\,500}{14\,700} = 272.14 \text{ mm} = 272 \text{ mm}$$ **Ans**

9-65. Locate the centroid $(\bar{x}, \bar{y})$ of the member's cross-sectional area.



Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	$A\ (mm^2)$	$\tilde{x}$ (mm)	$\tilde{y}$ (mm)	$\tilde{x}A\ (mm^3)$	$\tilde{y}A\ (mm^3)$
1	$\frac{1}{2}(30)(90)$	20	30	27 000	40 500
2	30(90)	45	45	121 500	121 500
3	100(50)	110	25	550 000	125 000
Σ	9 050			698 500	287 000

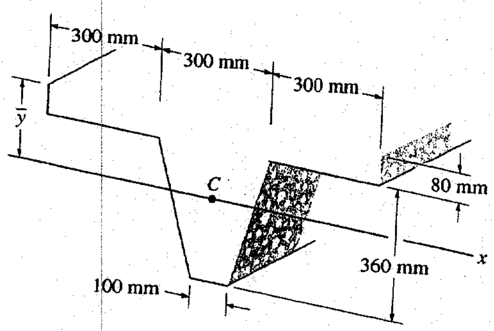


Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{698\ 500}{9\ 050} = 77.18\ \text{mm} = 77.2\ \text{mm} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{287\ 000}{9\ 050} = 31.71\ \text{mm} = 31.7\ \text{mm} \qquad \textbf{Ans}$$

9-66. Locate the centroid $\bar{y}$ of the concrete beam having the tapered cross section shown.

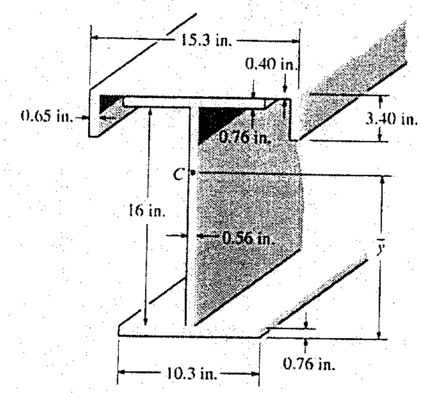


$$\Sigma \tilde{y}A = 900(80)(40) + 100(360)(260) + 2[\tfrac{1}{2}(100)(360)(200)] = 19.44(10^6)\ \text{mm}^3$$

$$\Sigma A = 900(80) + 100(360) + 2[\tfrac{1}{2}(100)(360)] = 0.144(10^6)\ \text{mm}^2$$

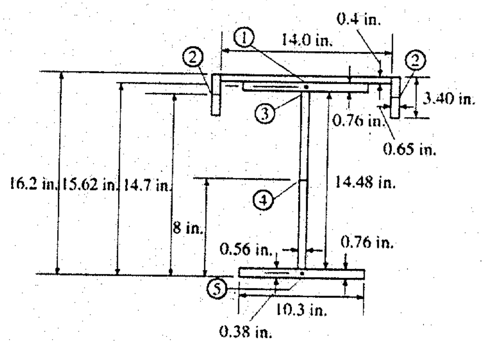
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{19.44(10^6)}{0.144(10^6)} = 135\ \text{mm} \qquad \textbf{Ans}$$

9-67. Locate the centroid $\bar{y}$ of the beam's cross-section built up from a channel and a wide-flange beam.



Centroid: The area of each segment and its respective centroid are tabulated below.

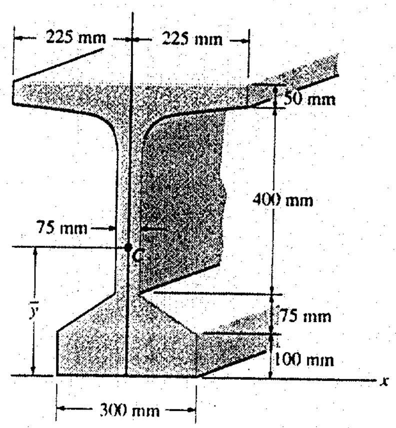
Segment	$A(\text{in}^2)$	$\tilde{y}(\text{in.})$	$\tilde{y}A(\text{in}^3)$
1	14.0(0.4)	16.20	90.72
2	3.40(1.30)	14.70	64.97
3	10.3(0.76)	15.62	122.27
4	14.48(0.56)	8.00	64.87
5	10.3(0.76)	0.38	2.97
Σ	33.78		345.81



Thus,

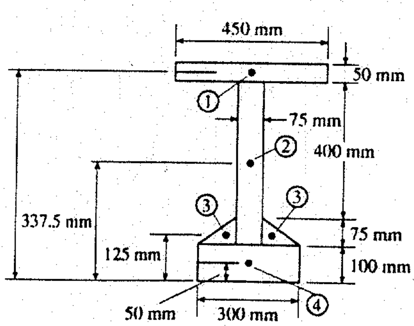
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{345.81}{33.78} = 10.24 \text{ in.} = 10.2 \text{ in.} \quad \textbf{Ans}$$

***9-68.** Locate the centroid $\bar{y}$ of the bulb-tee cross section.



Centroid: The area of each segment and its respective centroid are tabulated below.

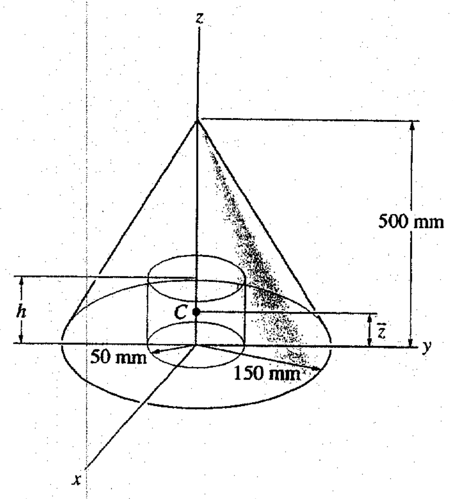
Segment	$A(\text{mm}^2)$	$\tilde{y}(\text{mm})$	$\tilde{y}A(\text{mm}^3)$
1	450(50)	600	13 500 000
2	475(75)	337.5	12 023 437.5
3	$\frac{1}{2}(225)(75)$	125	1 054 687.5
4	300(100)	50	1 500 000
Σ	96 562.5		28 078 125



Thus,

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{28\ 078\ 125}{96\ 562.5} = 290.78 \text{ mm} = 291 \text{ mm} \quad \textbf{Ans}$$

9-69. Determine the distance h to which a 100-mm diameter hole must be bored into the base of the cone so that the center of mass of the resulting shape is located at $\bar{z} = 115$ mm. The material has a density of 8 Mg/m³.



$$\frac{\frac{1}{3}\pi(0.15)^2(0.5)(\frac{0.5}{4}) - \pi(0.05)^2(h)(\frac{h}{2})}{\frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(h)} = 0.115$$

$$0.4313 - 0.2875\,h = 0.4688 - 1.25\,h^2$$

$$h^2 - 0.230\,h - 0.0300 = 0$$

Choosing the positive root,

$h = 323$ mm **Ans**

9-70. Determine the distance $\bar{z}$ to the centroid of the shape which consists of a cone with a hole of height $h = 50$ mm bored into its base.

$$\Sigma\tilde{z}V = \frac{1}{3}\pi(0.15)^2(0.5)(\frac{0.5}{4}) - \pi(0.05)^2(0.05)(\frac{0.05}{2})$$

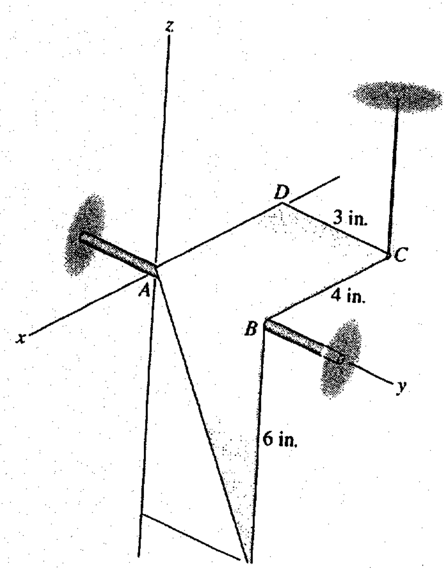
$$= 1.463(10^{-3})\ \text{m}^4$$

$$\Sigma V = \frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(0.05)$$

$$= 0.01139\ \text{m}^3$$

$$\bar{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{1.463(10^{-3})}{0.01139} = 0.12845\ \text{m} = 128\ \text{mm}$$ **Ans**

9-71. The sheet metal part has the dimensions shown. Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of its centroid.



$$\Sigma A = 4(3) + \frac{1}{2}(3)(6) = 21\ \text{in}^2$$

$$\Sigma\tilde{x}A = -2(4)(3) + 0\left(\frac{1}{2}\right)(3)(6) = -24\ \text{in}^3$$

$$\Sigma\tilde{y}A = 1.5(4)(3) + \frac{2}{3}(3)\left(\frac{1}{2}\right)(3)(6) = 36\ \text{in}^3$$

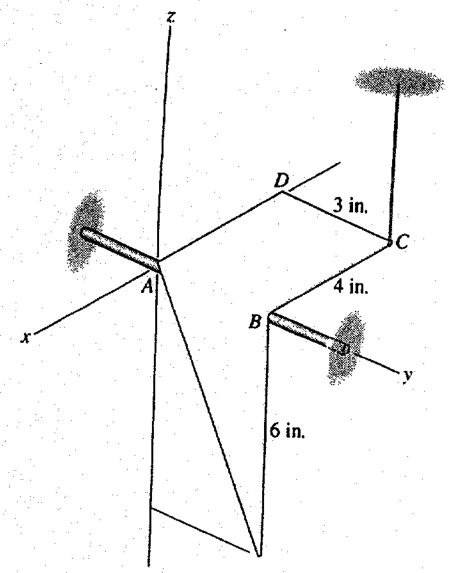
$$\Sigma\tilde{z}A = 0(4)(3) - \frac{1}{3}(6)\left(\frac{1}{2}\right)(3)(6) = -18\ \text{in}^3$$

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-24}{21} = -1.14\ \text{in.}$$ **Ans**

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{36}{21} = 1.71\ \text{in.}$$ **Ans**

$$\bar{z} = \frac{\Sigma\tilde{z}A}{\Sigma A} = \frac{-18}{21} = -0.857\ \text{in.}$$ **Ans**

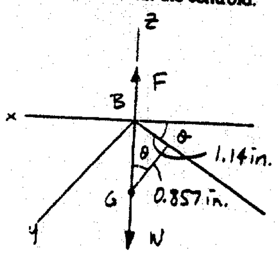
*9-72. The sheet metal part has a weight per unit area of 2 lb/ft^2 and is supported by the smooth rod and at C. If the cord is cut, the part will rotate about the y axis until it reaches equilibrium. Determine the equilibrium angle of tilt, measured downward from the negative x axis, that AD makes with the $-x$ axis.



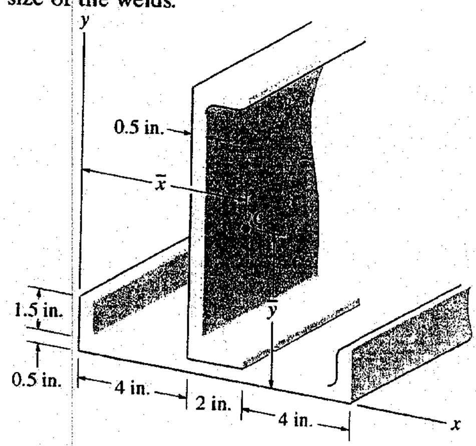
Since the material is homogeneous, the center of gravity coincides with the centroid.

See solution to Prob. 9-71.

$\theta = \tan^{-1}\left(\frac{1.14}{0.857}\right) = 53.1°$ **Ans**



9-73. Determine the location $(\bar{x}, \bar{y})$ of the centroid C of the cross-sectional area for the structural member constructed from two equal-sized channels welded together as shown. Assume all corners are square. Neglect the size of the welds.



$\Sigma\tilde{x}A = 1.5(0.5)(0.25) + 10(0.5)(5) + 1.5(0.5)(9.75) + 1.5(0.5)(5.25)(2) + 10(0.5)(4.25)$

$= 61.625 \text{ in}^3$

$\Sigma A = [1.5(0.5) + 10(0.5) + 1.5(0.5)](2) = 13 \text{ in}^2$

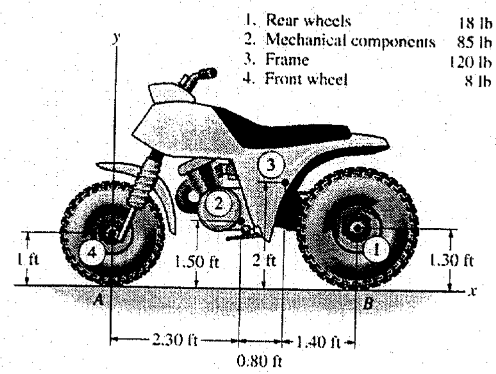
$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{61.625}{13} = 4.74 \text{ in.}$ **Ans**

$\Sigma\tilde{y}A = 1.5(0.5)(1.25)(2) + 10(0.5)(0.25) + 1.5(0.5)(0.75) + 10(0.5)(5.5) + 1.5(0.5)(10.25)$

$= 38.875 \text{ in}^3$

$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{38.875}{13} = 2.99 \text{ in.}$ **Ans**

9-74. Determine the location ($\bar{x}$, $\bar{y}$) of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the $x-y$ plane, determine the normal reactions each of its wheels exerts on the ground.



$$\Sigma\tilde{x}W = 4.5(18) + 2.3(85) + 3.1(120)$$

$$= 648.5 \text{ lb}\cdot\text{ft}$$

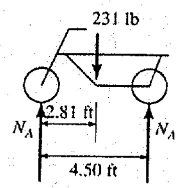
$$\Sigma W = 18 + 85 + 120 + 8 = 231 \text{ lb}$$

$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{648.5}{231} = 2.81 \text{ ft} \qquad \textbf{Ans}$$

$$\Sigma\tilde{y}W = 1.30(18) + 1.5(85) + 2(120) + 1(8)$$

$$= 398.9 \text{ lb}\cdot\text{ft}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{398.9}{231} = 1.73 \text{ ft} \qquad \textbf{Ans}$$



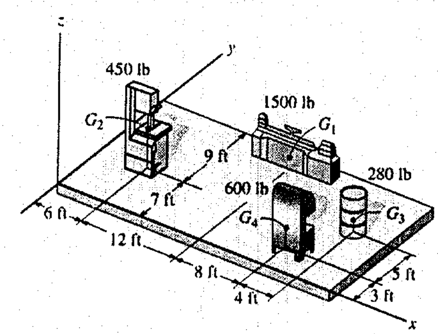
$$\circlearrowleft +\Sigma M_A = 0; \quad 2(N_B)(4.5) - 231(2.81) = 0$$

$$N_B = 72.1 \text{ lb} \qquad \textbf{Ans}$$

$$+\uparrow \Sigma F_y = 0; \quad N_A + 2(72.1) - 231 = 0$$

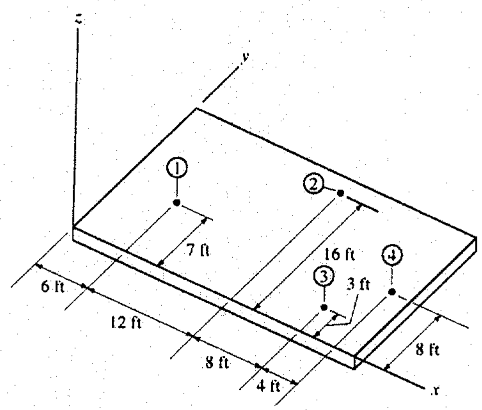
$$N_A = 86.9 \text{ lb} \qquad \textbf{Ans}$$

9-75. Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity G. Locate the center of gravity ($\bar{x}$, $\bar{y}$) of all these components.



Centroid: The floor loadings on the floor and its respective centroid are tabulated below.

Loading	W (lb)	$\tilde{x}$ (ft)	$\tilde{y}$ (ft)	$\tilde{x}W$ (lb·ft)	$\tilde{y}W$ (lb·ft)
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
Σ	2830			53700	31190

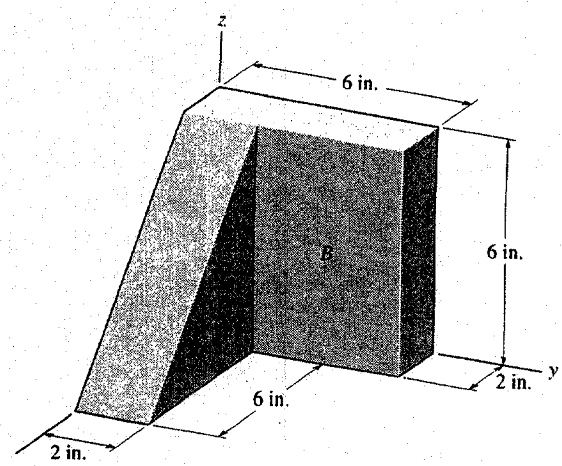


Thus,

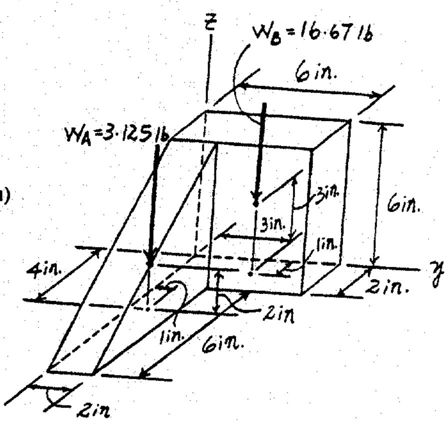
$$\bar{x} = \frac{\Sigma\tilde{x}W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma\tilde{y}W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \qquad \textbf{Ans}$$

***9-76.** Locate the center of gravity of the two-block assembly. The specific weights of the materials A and B are $\gamma_A = 150\ \text{lb/ft}^3$ and $\gamma_B = 400\ \text{lb/ft}^3$, respectively.



Centroid : The weight of block A and B are $W_A = \frac{1}{2}\left(\frac{6}{12}\right)\left(\frac{6}{12}\right)\left(\frac{2}{12}\right)(150) = 3.125$ lb and $W_B = \left(\frac{6}{12}\right)\left(\frac{6}{12}\right)\left(\frac{2}{12}\right)(400) = 16.67$ lb. The weight of each block and its respective centroid are tabulated below.



Block	W (lb)	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{z}$ (in.)	$\tilde{x}W$ (lb·in)	$\tilde{y}W$ (lb·in)	$\tilde{z}W$ (lb·in)
A	3.125	4	1	2	12.5	3.125	6.25
B	16.667	1	3	3	16.667	50.0	50.0
Σ	19.792				29.167	53.125	56.25

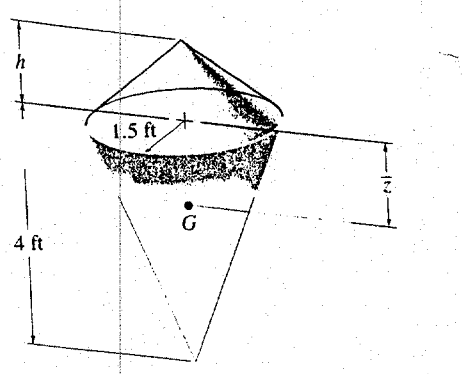
Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}W}{\Sigma W} = \frac{29.167}{19.792} = 1.474 \text{ in.} = 1.47 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{y} = \frac{\Sigma \tilde{y}W}{\Sigma W} = \frac{53.125}{19.792} = 2.684 \text{ in.} = 2.68 \text{ in.} \qquad \textbf{Ans}$$

$$\bar{z} = \frac{\Sigma \tilde{z}W}{\Sigma W} = \frac{56.25}{19.792} = 2.842 \text{ in.} = 2.84 \text{ in.} \qquad \textbf{Ans}$$

9-77. The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If $h = 1.2$ ft, find the distance $\bar{z}$ to the buoy's center of gravity G.



$$\Sigma \tilde{z} V = \frac{1}{3}\pi(1.5)^2(1.2)\left(-\frac{1.2}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

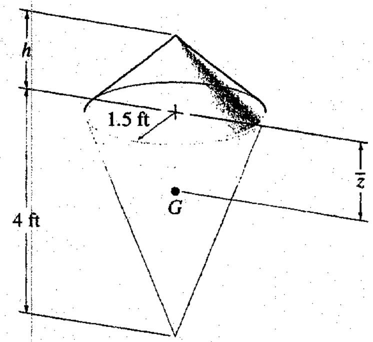
$$= 8.577 \text{ ft}^4$$

$$\Sigma V = \frac{1}{3}\pi(1.5)^2(1.2) + \frac{1}{3}\pi(1.5)^2(4)$$

$$= 12.25 \text{ ft}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{8.577}{12.25} = 0.70 \text{ ft} \qquad \textbf{Ans}$$

9-78. The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If it is required that the buoy's center of gravity G be located at $\bar{z} = 0.5$ ft, determine the height h of the top cone.



$$\Sigma \tilde{z} V = \frac{1}{3}\pi(1.5)^2(h)\left(-\frac{h}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$
$$= -0.5890\,h^2 + 9.4248$$

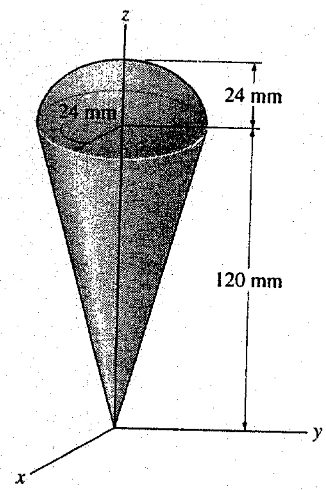
$$\Sigma V = \frac{1}{3}\pi(1.5)^2(h) + \frac{1}{3}\pi(1.5)^2(4)$$
$$= 2.3562\,h + 9.4248$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{-0.5890\,h^2 + 9.4248}{2.3562\,h + 9.4248} = 0.5$$

$$-0.5890\,h^2 + 9.4248 = 1.1781\,h + 4.7124$$

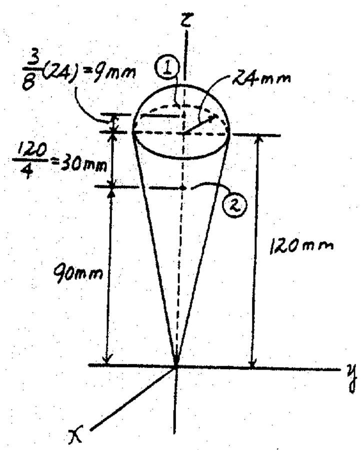
$h = 2.00$ ft **Ans**

9-79. Locate the centroid $\bar{z}$ of the top made from a hemisphere and a cone.



Centroid : The volume of each segment and its respective centroid are tabulated below.

Segment	$V\,(\text{mm}^3)$	$\tilde{z}\,(\text{mm})$	$\tilde{z}V\,(\text{mm}^4)$
1	$\frac{2}{3}\pi(24^3)$	129	$1.188864\pi(10^6)$
2	$\frac{1}{3}\pi(24^2)(120)$	90	$2.0736\pi(10^6)$
Σ	$32.256\pi(10^3)$		$3.262464\pi(10^6)$



Thus,

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{3.262464\pi(10^6)}{32.256\pi(10^3)} = 101.14 \text{ mm} = 101 \text{ mm}$$ **Ans**