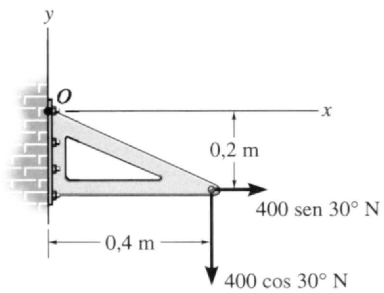
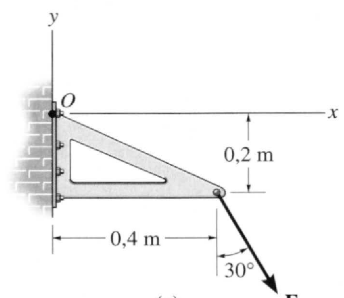


(b)



(c)

Figura 4.20

ou

$$\mathbf{M}_O = \{-98{,}6\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textit{Resposta}$$

SOLUÇÃO II (ANÁLISE VETORIAL)

Utilizando a aproximação de vetor cartesiano, os vetores força e posição mostrados na Figura 4.20*c* podem ser representados como:

$$\mathbf{r} = \{0{,}4\mathbf{i} - 0{,}2\mathbf{j}\}\ \text{m}$$

$$\mathbf{F} = \{400 \text{ sen } 30°\mathbf{i} - 400 \cos 30°\mathbf{j}\}\ \text{N}$$

$$= \{200\mathbf{i} - 346{,}4\mathbf{j}\}\ \text{N}$$

O momento é, portanto:

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0{,}4 & -0{,}2 & 0 \\ 200 & -346{,}4 & 0 \end{vmatrix}$$

$$= 0\mathbf{i} - 0\mathbf{j} + [0{,}4(-346{,}4) - (-0{,}2)(200)]\mathbf{k}$$

$$= \{-98{,}6\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textit{Resposta}$$

Comparando, percebe-se que a análise escalar (solução I) forneceu um *procedimento mais conveniente* para análise do que a solução II, uma vez que a direção e o sentido do momento, bem como os braços de momento para cada componente de força, foram facilmente determinados. Por isso, esse método costuma ser recomendado para a resolução de problemas que envolvem duas dimensões. Já a análise de vetores cartesianos em geral é recomendada somente para a solução de problemas tridimensionais, uma vez que os braços de momento e os componentes das forças são freqüentemente mais difíceis de determinar.

PROBLEMAS

4.1. Sendo **A**, **B** e **D** vetores conhecidos, prove a propriedade distributiva para o produto vetorial, isto é, $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

4.2. Prove a identidade com o produto vetorial tríplice $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \times \mathbf{B}) \cdot \mathbf{C}$.

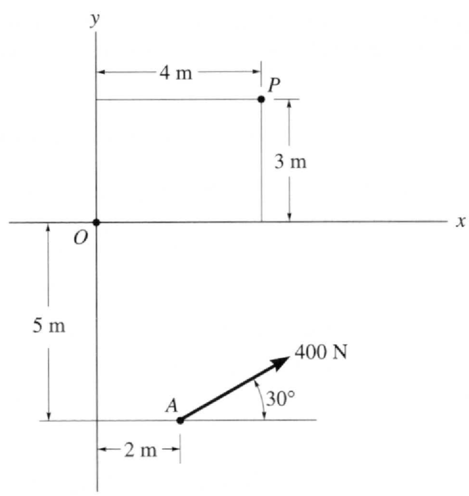
4.3. Dados três vetores não-nulos **A**, **B** e **C**, mostre que se $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, então os três vetores *devem* ser coplanares.

***4.4.** Determine a intensidade, a direção e o sentido do momento da força em *A* em relação ao ponto *O*.

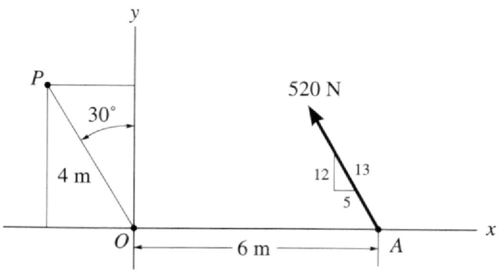
4.5. Determine a intensidade, a direção e o sentido do momento da força em *A* em relação a um ponto *P*.

4.6. Determine a intensidade, a direção e o sentido do momento da força em *A* em relação ao ponto *O*.

4.7. Determine a intensidade, a direção e o sentido do momento da força em *A* em relação a um ponto *P*.



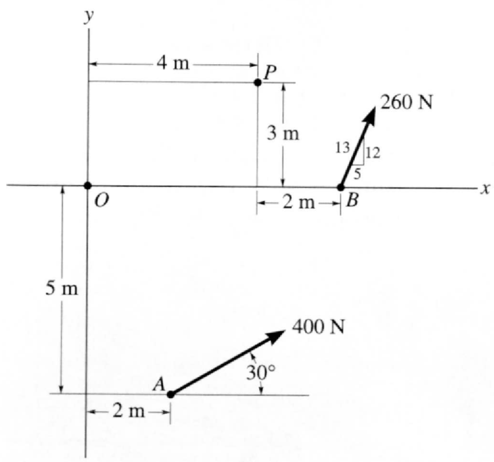
Problemas 4.4/5



Problemas 4.6/7

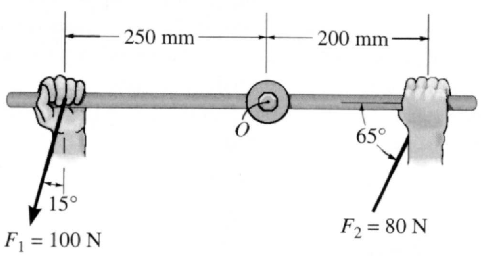
***4.8.** Determine a intensidade, a direção e o sentido do momento resultante das forças em relação ao ponto *O*.

4.9. Determine a intensidade, a direção e o sentido do momento resultante das forças em relação ao ponto *P*.



Problemas 4.8/9

4.10. A chave de boca é usada para soltar o parafuso. Determine o momento de cada força em relação ao eixo do parafuso que passa através do ponto *O*.



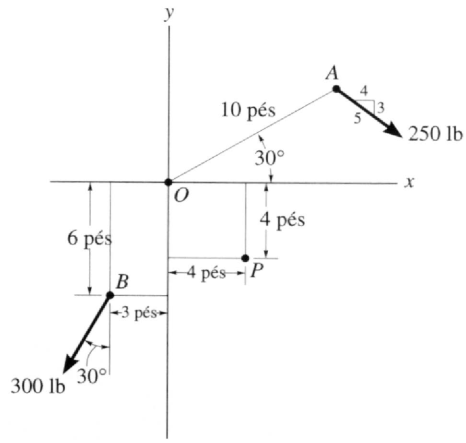
Problema 4.10

4.11. Determine a intensidade, a direção e o sentido do momento resultante das forças em relação ao ponto *O*.

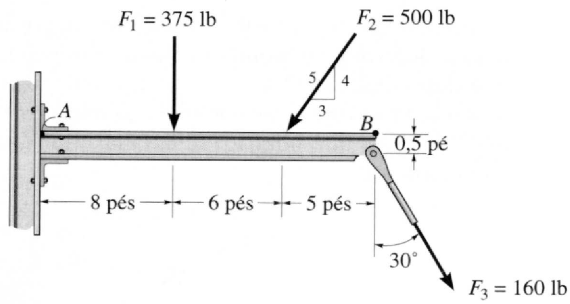
***4.12.** Determine o momento em relação ao ponto *A* de cada uma das três forças agindo sobre a viga.

4.13. Determine o momento em relação ao ponto *B* de cada uma das três forças que atuam na viga.

4.14. Determine o momento de cada força em relação ao parafuso localizado em *A*. Considere $F_B = 40$ lb e $F_C = 50$ lb.

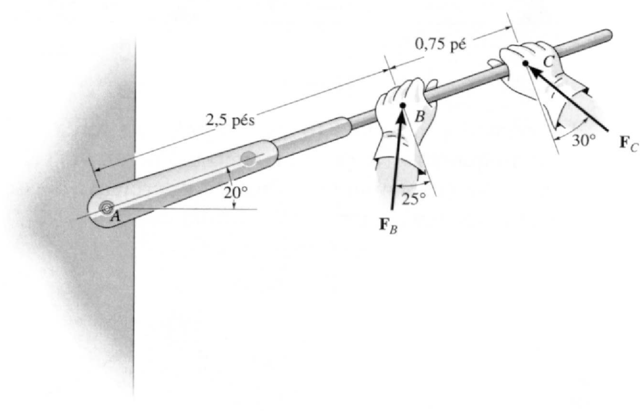


Problema 4.11



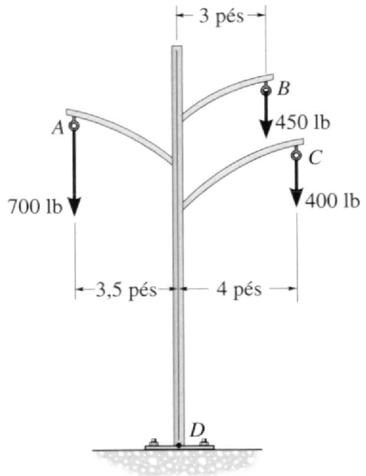
Problemas 4.12/13

4.15. Se $F_B = 30$ lb e $F_C = 45$ lb, determine o momento resultante em relação ao parafuso localizado em *A*.



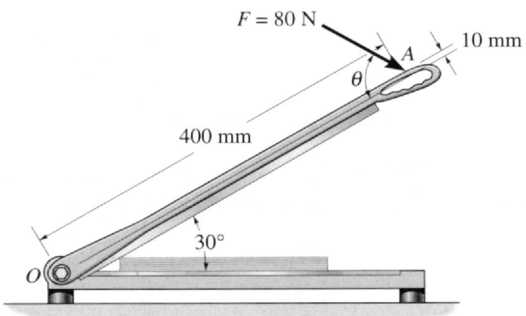
Problemas 4.14/15

***4.16.** O poste de energia elétrica suporta as três linhas. Cada linha exerce uma força vertical sobre o poste devido ao próprio peso, conforme mostra a figura. Determine o momento resultante na base *D* provocado por todas essas forças. Supondo que seja possível que o vento ou o gelo sejam capazes de romper as linhas, determine qual ou quais linhas, quando rompidas, criariam a condição para o máximo momento em relação à base. Qual será esse momento resultante?



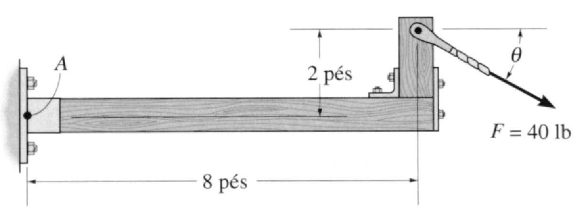
Problema 4.16

4.17. Uma força de 80 N atua sobre o cabo de um cortador de papel em A. Determine o momento criado por essa força em relação à dobradiça em O, se $\theta = 60°$. Em que ângulo θ a força deve ser aplicada para que o momento criado em relação ao ponto O (no sentido horário) seja o máximo? Qual é esse máximo momento?



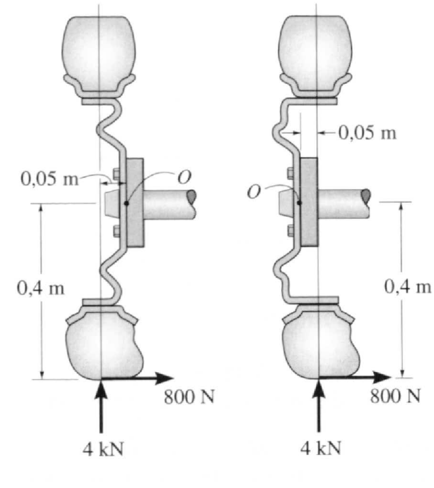
Problema 4.17

4.18. Determine a direção θ ($0° \leq \theta \leq 180°$) da força F = 40 lb de modo que ela crie (a) o máximo momento em relação ao ponto A e (b) o mínimo momento em relação a esse mesmo ponto. Calcule o momento em cada caso.



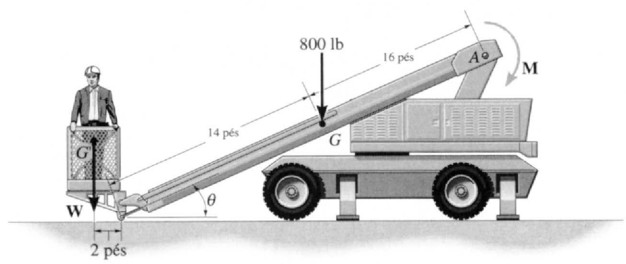
Problema 4.18

4.19. O cubo de roda na figura pode ser fixado ao eixo tanto com um afastamento negativo (para a esquerda) como com um afastamento positivo (para a direita). Se o pneu está sujeito às cargas normal e radial, como mostrado, determine o momento resultante dessas cargas em relação ao eixo no ponto O em ambos os casos.



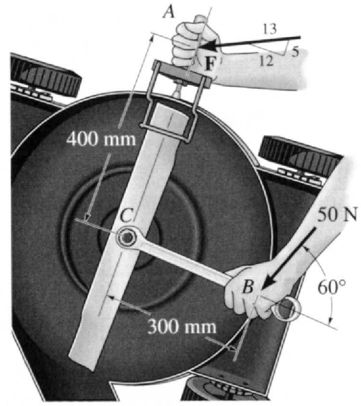
Problema 4.19

***4.20.** O braço da grua tem comprimento de 30 pés, peso de 800 lb e centro de massa em G. Se o máximo momento que pode ser desenvolvido pelo motor em A é $M = 20 \times 10^3$ lb · pés, determine a máxima carga W, com centro de massa em G', que pode ser elevada. Considere $\theta = 30°$.



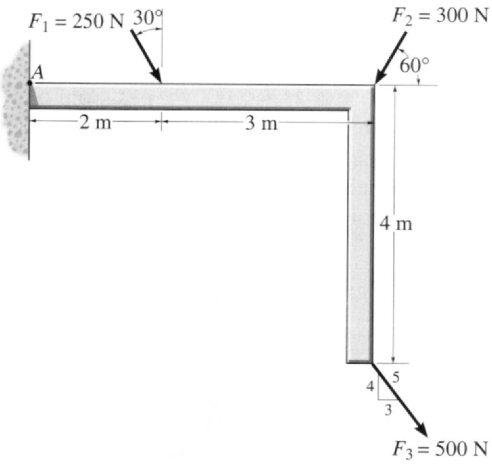
Problema 4.20

4.21. A ferramenta em A é usada para prender uma lâmina estacionária de cortador de grama, enquanto a porca é solta com uma chave. Se a força de 50 N é aplicada à chave em B na direção e no sentido mostrados na figura, determine o momento criado em relação à porca em C. Qual é a intensidade da força **F** em A de modo a gerar o momento oposto em relação a C?



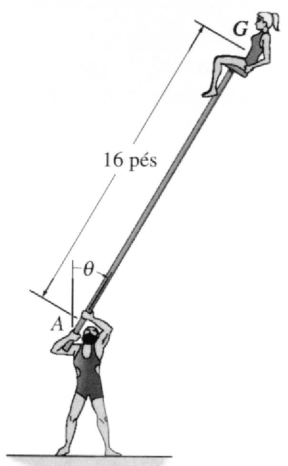
Problema 4.21

4.22. Determine o momento de cada uma das três forças em relação ao ponto A. Resolva o problema primeiro utilizando cada força como um todo e, depois, o princípio dos momentos.



Problema 4.22

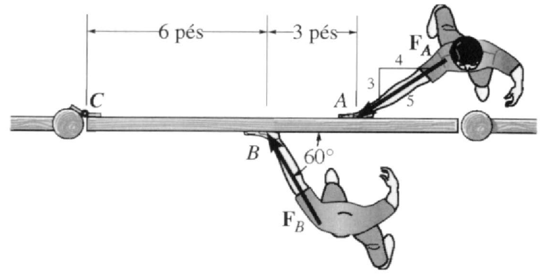
4.23. Como parte de uma manobra acrobática, um homem sustenta uma garota que pesa 120 lb e está sentada em uma cadeira no alto de um mastro. Estando o centro de gravidade da garota localizado em G e sendo de 250 lb · pés o máximo momento no sentido horário que o homem pode exercer sobre o mastro no ponto A, determine o ângulo máximo de inclinação, θ, que não permite que a garota caia, isto é, que seu momento anti-horário em relação ao ponto A não seja maior do que 250 lb · pés.



Problema 4.23

*__4.24.__ Os dois garotos empurram o portão com forças de $F_A = 30$ lb e $F_B = 50$ lb, como mostra a figura. Determine o momento de cada força em relação a C. O portão sofrerá uma rotação no sentido horário ou anti-horário? Despreze a espessura do portão.

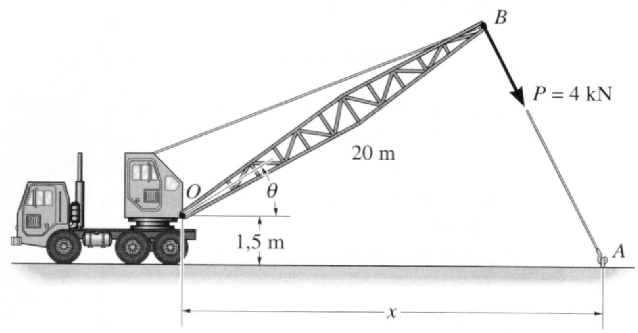
4.25. Se o garoto aplica em B uma força $F_B = 30$ lb, determine a intensidade da força F_A que ele deve aplicar em A a fim de evitar que o portão gire. Despreze a espessura do portão.



Problemas 4.24/25

4.26. O cabo do reboque exerce uma força $P = 4$ kN na extremidade do guindaste de 20 m de comprimento. Se $\theta = 30°$, determine o valor de x do gancho preso em A, de forma que essa força crie um momento máximo em relação ao ponto O. Nessa condição, qual é esse momento?

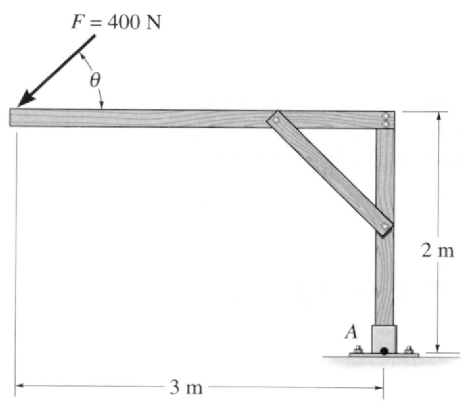
4.27. O cabo do reboque aplica uma força $P = 4$ kN na extremidade do guindaste de 20 m de comprimento. Sendo $x = 25$ m, determine a posição θ do guindaste, de modo que a força crie um momento máximo em relação ao ponto O. Qual é esse momento?



Problemas 4.26/27

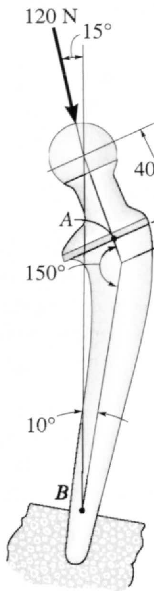
*__4.28.__ Determine a direção θ, com $0° \leq \theta \leq 180°$, da força **F**, de maneira que ela produza (a) o máximo momento em relação ao ponto A e (b) o mínimo momento em relação ao ponto A. Calcule o momento em cada caso.

4.29. Determine o momento da força **F** em relação ao ponto A como uma função de θ. Faça um gráfico do resultado com M (na ordenada) e θ (na abscissa) para $0° \leq \theta \leq 180°$.



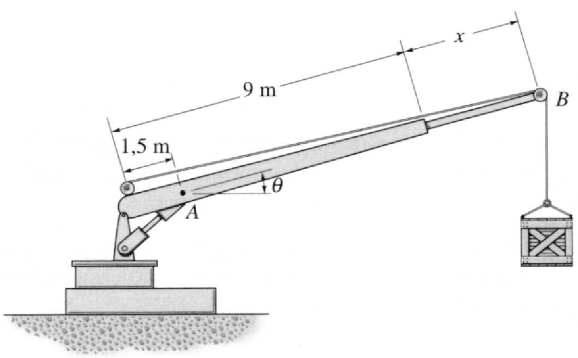
Problemas 4.28/29

4.30. A prótese do quadril está sujeita à força F = 120 N. Determine o momento dessa força em relação ao pescoço em A e à haste em B.



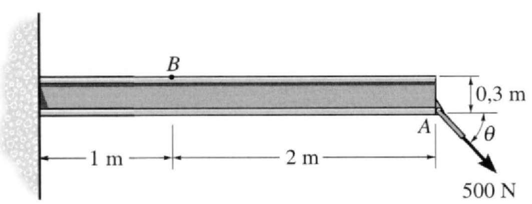
Problema 4.30

4.31. O guindaste pode ser ajustado para qualquer ângulo $0° \leq \theta \leq 90°$ e qualquer extensão $0 \leq x \leq 5$ m. Para uma massa suspensa de 120 kg, determine o momento desenvolvido em A como função de x e θ. Quais valores de x e θ conduzem ao máximo momento possível em A? Calcule esse momento. Despreze as dimensões da polia em B.



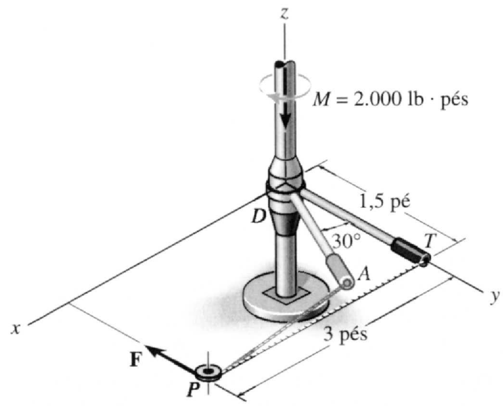
Problema 4.31

***4.32.** Determine o ângulo θ para o qual a força de 500 N deve atuar em A para que o momento dessa força em relação ao ponto B seja igual a zero.



Problema 4.32

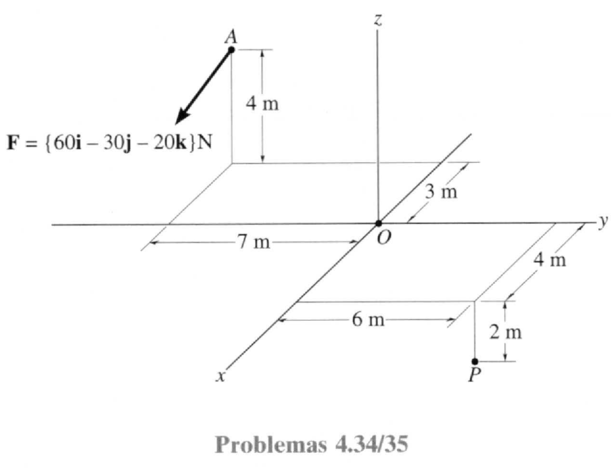
4.33. Segmentos de um tubo D para perfuração de um poço de petróleo estão ajustados por meio de uma pinça reguladora T que aperta o tubo e de um cilindro hidráulico (não mostrado na figura), para regular a força **F** aplicada à pinça. Essa força atua ao longo do cabo que passa ao redor de uma pequena polia P. Estando o cabo originalmente perpendicular à pinça, como mostrado na figura, determine a intensidade da força **F** que deve ser aplicada de modo que o momento em relação ao tubo seja M = 2.000 lb·pés. Com o intuito de manter esse mesmo momento, qual intensidade de **F** é necessária quando a pinça é ajustada em 30°, como na posição esboçada com tonalidade mais clara? *Nota:* o ângulo DAP não é 90° nessa posição.



Problema 4.33

4.34. Determine o momento de uma força no ponto A em relação ao ponto O. Expresse o resultado como um vetor cartesiano.

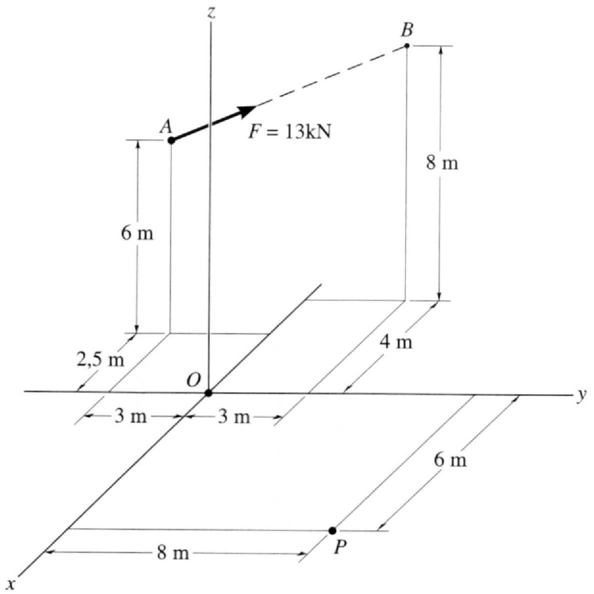
4.35. Determine o momento da força em A em relação ao ponto P. Expresse o resultado como um vetor cartesiano.



Problemas 4.34/35

***4.36.** Determine o momento da força **F** em A relativamente ao ponto O. Expresse o resultado como um vetor cartesiano.

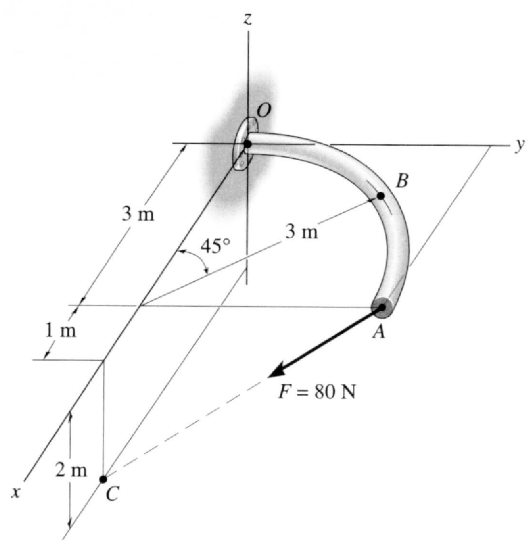
4.37. Determine o momento da força **F** no ponto A em relação ao ponto P. Expresse o resultado como um vetor cartesiano.



Problemas 4.36/37

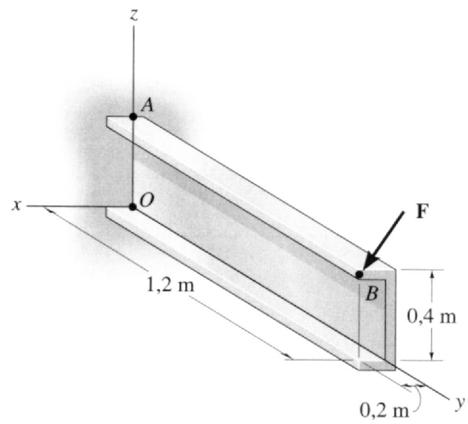
4.38. O bastão curvado se estende no plano $x-y$ e tem um raio de curvatura de 3 m. Se a força $F = 80$ N atua em sua extremidade, como é mostrado na figura, determine o momento dessa força em relação ao ponto O.

4.39. O bastão curvado se estende no plano $x-y$ e tem um raio de curvatura de 3 m. Se a força $F = 80$ N atua em sua extremidade, como é mostrado na figura, determine o momento dessa força em relação ao ponto B.



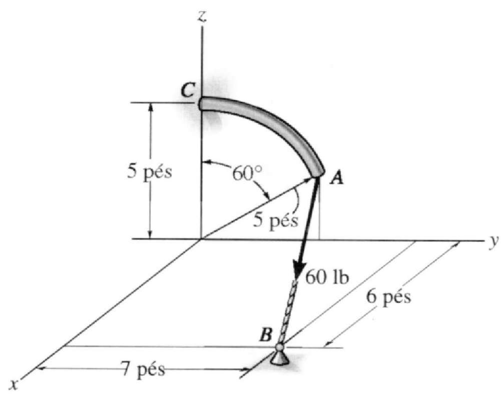
Problemas 4.38/39

***4.40.** A força $\mathbf{F} = \{600\mathbf{i} + 300\mathbf{j} - 600\mathbf{k}\}$ N atua na extremidade da viga. Determine o momento da força em relação ao ponto A.



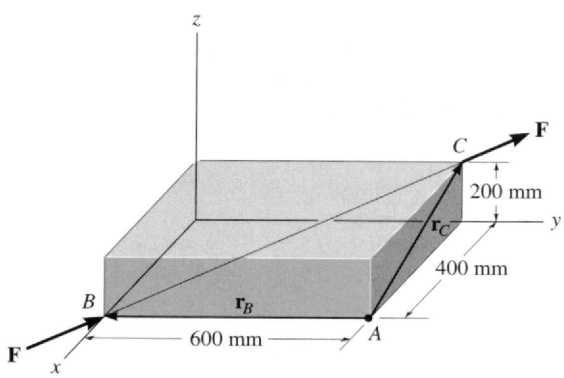
Problema 4.40

4.41. O bastão curvado tem raio de curvatura de 5 pés. Se uma força de 60 lb atua em sua extremidade, como mostrado na figura, determine o momento dessa força em relação a C.



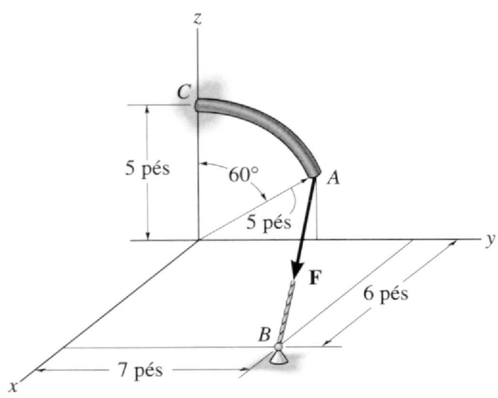
Problema 4.41

4.42. Uma força $\mathbf{F}$ com intensidade $F = 100$ N atua ao longo da diagonal do paralelepípedo. Determine o momento de $\mathbf{F}$ em relação ao ponto A, utilizando $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ e $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$.



Problema 4.42

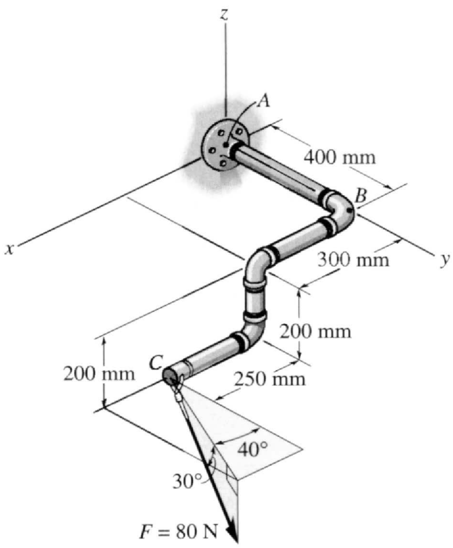
4.43. Determine a menor força F que deve ser aplicada na corda para envergar o bastão, o qual tem raio de 5 pés, até que ele quebre no suporte C. Isso requer que o ponto C sofra um momento $M = 80$ lb·pés.



Problema 4.43

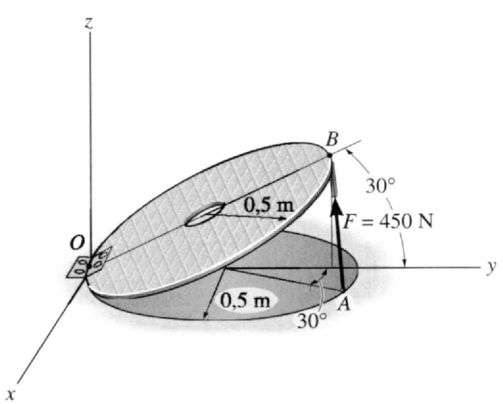
*__4.44.__ A estrutura tubular da figura está sujeita à força de 80 N. Determine o momento dessa força em relação ao ponto A.

4.45. Agora, determine o momento dessa força em relação ao ponto B.



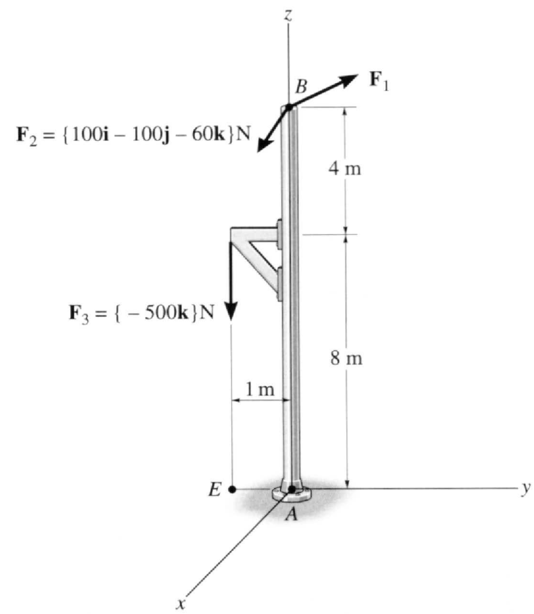
Problemas 4.44/45

4.46. A escora AB de uma comporta de 1 m de diâmetro exerce uma força de 450 N no ponto B. Determine o momento dessa força em relação ao ponto O.



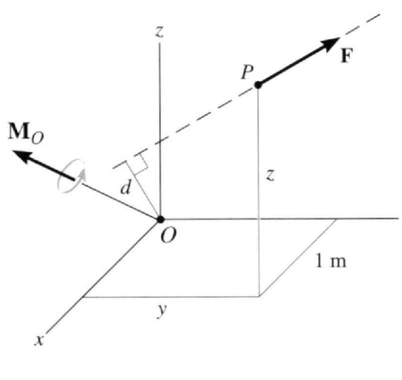
Problema 4.46

4.47. Usando a análise vetorial cartesiana, determine o momento resultante das três forças em relação à base da coluna em A, dado: $\mathbf{F}_1 = \{400\mathbf{i} + 300\mathbf{j} + 120\mathbf{k}\}$ N.



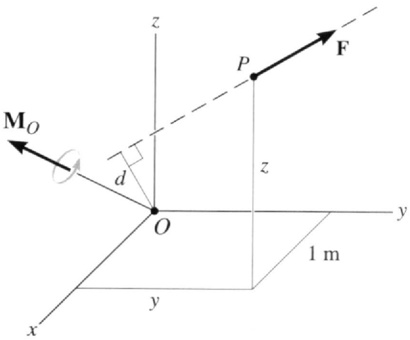
Problema 4.47

*__4.48.__ Uma força $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produz um momento $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m em relação à origem das coordenadas no ponto O. Considerando que a força atua em um ponto com coordenadas $x = 1$ m, determine as demais coordenadas y e z.



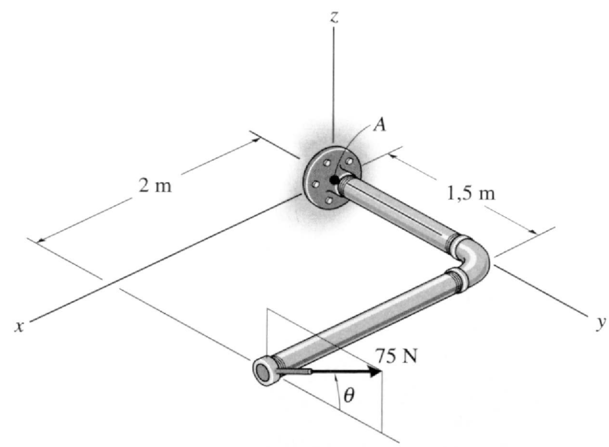
Problema 4.48

4.49. A força $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N dá origem a um momento em relação ao ponto O de $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$ N·m. Considerando que a força atua em um ponto com coordenada x igual a 1 m, determine as coordenadas y e z desse ponto. Além disso, considere que $M_O = Fd$ e encontre a distância perpendicular d do ponto O até a linha de ação da força $\mathbf{F}$.



Problema 4.49

■**4.50.** Usando uma peça anelar, a força de 75 N pode ser aplicada no plano vertical para vários ângulos θ. Determine a intensidade do momento produzido em relação ao ponto A. Faça um gráfico do resultado de M (na ordenada) *versus* θ (na abscissa) para $0° \leq \theta \leq 180°$ e especifique os ângulos que fornecem os momentos máximo e mínimo.



Problema 4.50

4.5 MOMENTO DE UMA FORÇA EM RELAÇÃO A UM EIXO ESPECÍFICO

Lembre-se de que, quando o momento de uma força é calculado em relação a um ponto, seu eixo é *sempre* perpendicular ao plano contendo a força e o braço do momento. Em alguns problemas, é importante encontrar *o componente* desse momento ao longo de um *eixo específico* que passa pelo ponto. Na resolução desses problemas, pode ser usada a análise escalar ou a vetorial.

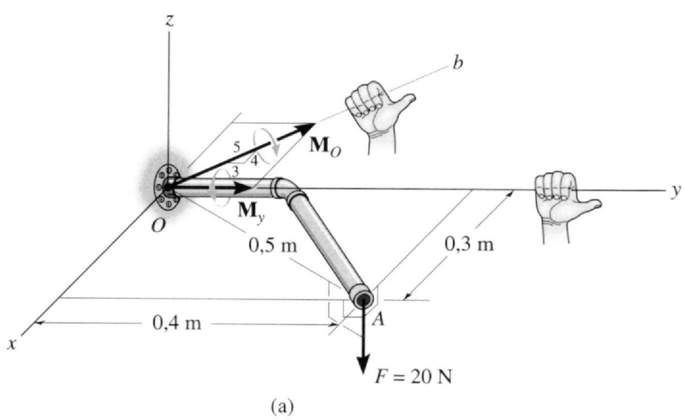


Figura 4.21

Análise Escalar. Para mostrar a resolução numérica desse tipo de problema, considere a estrutura tubular apresentada na Figura 4.21*a*, que se estende no plano horizontal e está sujeita à força vertical $F = 20$ N aplicada no ponto A. O momento dessa força em relação ao ponto O tem a *intensidade* dada por $M_O = (20\text{ N})\,(0{,}5\text{ m}) = 10\text{ N}\cdot\text{m}$, com *direção e sentido* definidos pela regra da mão direita, como mostra a Figura 4.21*a*. Esse momento tende a girar o conjunto em relação ao eixo Ob. Por razões práticas, no entanto, pode ser necessário determinar *o componente* de $\mathbf{M}_O$ em relação ao eixo y, $\mathbf{M}_y$, uma vez que esse

3.15. $F = 158$ N
3.17. $W = 76{,}6$ lb
3.18. $\theta = 78{,}7°$, $F_{CD} = 127$ lb
3.19. $\theta = 78{,}7°$, $W = 51{,}0$ lb
3.21. $d = 2{,}42$ m
3.22. $\theta = 60°$, $T_{AB} = 34{,}6$ lb
3.23. $\theta = 60°$, $W = 46{,}2$ lb
3.25. $s = 5{,}33$ pés
3.26. $W = 6$ lb
3.27. $F_{AC} = F_{AB} = F = \{2{,}45 \text{ cosec } \theta\}$ kN, $l = 1{,}72$ m
3.29. $l = 19{,}1$ pol
3.30. Em C e D, $T = 106$ lb
3.31. $\theta = 35{,}0°$
3.33. $W_B = 18{,}3$ lb
3.34. $l = 2{,}65$ pés
3.35. $F_{BD} = 171$ N, $F_{BC} = 145$ N
3.37. $\theta = 43{,}0°$
3.38. $y = 6{,}59$ m
3.39. $m_B = 3{,}58$ kg, $N = 19{,}7$ N
3.41. $F_1 = 608$ N, $\alpha = 79{,}2°$, $\beta = 16{,}4°$, $\gamma = 77{,}8°$
3.42. $F_1 = 800$ N, $F_2 = 147$ N, $F_3 = 564$ N
3.43. $F_1 = 5{,}60$ kN, $F_2 = 8{,}55$ kN, $F_3 = 9{,}44$ kN
3.45. $F_{AD} = 1{,}20$ kN, $F_{AC} = 0{,}40$ kN, $F_{AB} = 0{,}80$ kN
3.46. $F_{AC} = 130$ N, $F_{AD} = 510$ N, $F = 1{,}06$ kN
3.47. $s_{OB} = 327$ mm, $s_{OA} = 218$ mm
3.49. $F_{AB} = 0{,}980$ kN, $F_{AC} = 0{,}463$ kN, $F_{AD} = 1{,}55$ kN
3.50. $F_{AO} = 319$ N, $F_{AB} = 110$ N, $F_{AC} = 85{,}8$ N
3.51. $W = 138$ N
3.53. $F_{AE} = F_{AD} = 1{,}42$ kN $F_{AB} = 1{,}32$ kN
3.54. $F_{AB} = F_{AC} = 16{,}6$ kN, $F_{AD} = 55{,}2$ kN
3.55. $F_B = 19{,}2$ kN, $F_C = 10{,}4$ kN, $F_D = 6{,}32$ kN
3.57. $F_{AB} = 520$ N, $F_{AC} = F_{AD} = 260$ N, $d = 3{,}61$ m
3.58. $F_{AB} = 35{,}9$ lb, $F_{AC} = F_{AD} = 25{,}4$ lb
3.59. $W = 267$ lb
3.61. $F_{AB} = 469$ lb, $F_{AC} = F_{AD} = 331$ lb
3.62. $x = 0{,}190$ m, $y = 0{,}0123$ m
3.63. $F_{AD} = 1{,}42$ kip, $F_{AC} = 0{,}914$ kip, $F_{AB} = 1{,}47$ kip
3.65. $F_{OB} = 120$ N, $F_{OC} = 150$ N, $F_{OD} = 480$ N
3.66. $F_A = 34{,}6$ lb, $F_B = 57{,}3$ lb
3.67. $F = 40{,}8$ lb
3.69. Romeu pode subir pela corda.
Romeu e Julieta podem descer pela corda.
3.70. $F_1 = 8{,}26$ kN, $F_2 = 3{,}84$ kN, $F_3 = 12{,}2$ kN
3.71. $\theta = 90°$, $F_{AC} = 160$ lb, $\theta = 120°$, $F_{AB} = 160$ lb
3.73. $W = 240$ lb
3.74. $F_{CD} = 625$ lb, $F_{CA} = F_{CB} = 198$ lb
3.75. $F_1 = 0$, $F_2 = 311$ lb, $F_3 = 238$ lb

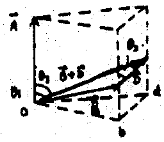
Capítulo 4

4.3. Se $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, então o volume é igual a zero, de modo que **A**, **B** e **C** são coplanares.
4.5. $M_P = 2{,}37$ kN·m ↺
4.6. $M_O = 2{,}88$ kN·m ↻
4.7. $M_P = 3{,}15$ kN·m ↻
4.9. $M_P = 3{,}15$ kN·m ↺
4.10. $(M_{F_1})_O = 24{,}1$ N·m ↻,
$(M_{F_2})_O = 14{,}5$ N·m ↻,
4.11. $M_O = 2{,}42$ kip·pés ↻
4.13. $(M_{F_1})_B = 4{,}125$ kip·pés ↻,
$(M_{F_2})_B = 2{,}00$ kip·pés ↻,
$(M_{F_3})_B = 40{,}0$ lb·pés ↻
4.14. $M_B = 90{,}6$ lb·pés ↺, $M_C = 141$ lb·pés ↺
4.15. $M_A = 195$ lb·pés ↺
4.17. $M_O = 28{,}1$ N·m ↻, $\theta = 88{,}6°$,
$(M_O)_{máx} = 32{,}0$ N·m ↻
4.18. **a)** $(M_A)_{máx} = 330$ lb·pés, $\theta = 76{,}0°$,
b) $(M_A)_{mín} = 0$, $\theta = 166°$
4.19. $-M_O = 120$ N·m ↻, $+M_O = 520$ N·m ↻
4.21. **a)** $M_A = 13{,}0$ N·m ↻, **b)** $F = 35{,}2$ N
4.22. $(M_{F_1})_A = 433$ N·m ↻,
$(M_{F_2})_A = 1{,}30$ kN·m ↻,
$(M_{F_3})_A = 800$ N·m ↻,
4.23. $\theta = 7{,}48°$
4.25. $F_A = 28{,}9$ lb
4.26. $(M_O)_{máx} = 80$ kN·m, $x = 24{,}0$ m
4.27. $(M_O)_{máx} = 80{,}0$ kN·m, $\theta = 33{,}6°$
4.29. $M_A = 1200 \text{ sen } \theta + 800 \cos \theta$ ↻
4.30. $M_A = 0{,}418$ N·m ↻,
$M_B = 4{,}92$ N·m ↻
4.31. $M_A = \{1{,}18 \cos \theta(7{,}5 + x)\}$ kN·m ↻,
$(M_A)_{máx} = 14{,}7$ kN·m ↻
4.33. $F = 1{,}33$ kip, $F' = 1{,}63$ kip
4.34. $\mathbf{M}_O = \{260\mathbf{i} + 180\mathbf{j} + 510\mathbf{k}\}$ N·m
4.35. $\mathbf{M}_O = \{440\mathbf{i} + 220\mathbf{j} + 990\mathbf{k}\}$ N·m
4.37. $\mathbf{M}_P = \{-116\mathbf{i} + 16\mathbf{j} - 135\mathbf{k}\}$ kN·m
4.38. $\mathbf{M}_O = \{-128\mathbf{i} + 128\mathbf{j} - 257\mathbf{k}\}$ N·m
4.39. $\mathbf{M}_B = \{-37{,}6\mathbf{i} + 90{,}7\mathbf{j} - 155\mathbf{k}\}$ N·m
4.41. $\mathbf{M}_C = \{-35{,}4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\}$ lb·pés
4.42. $\mathbf{M}_A = \{-16{,}0\mathbf{i} - 32{,}1\mathbf{k}\}$ N·m
4.43. $F_{AB} = 18{,}6$ lb
4.45. $\mathbf{M}_B = \{10{,}6\mathbf{i} + 13{,}1\mathbf{j} + 29{,}2\mathbf{k}\}$ N·m
4.46. $\mathbf{M}_O = \{373\mathbf{i} - 99{,}9\mathbf{j} + 173\mathbf{k}\}$ N·m
4.47. $\mathbf{M}_R = \{-1{,}90\mathbf{i} + 6{,}00\mathbf{j}\}$ kN·m
4.49. $y = 1$ m, $z = 3$ m, $d = 1{,}15$ m
4.50. $M_A = \sqrt{12\,656{,}25 \text{ sen}^2 \theta + 22\,500}$,
$M_{máx}$ em $\theta = 90°$, $M_{mín}$ em $\theta = 0°, 180°$
4.51. $(\mathbf{M}_{Oa})_P = \{218\mathbf{j} + 163\mathbf{k}\}$ N·m
4.53. $(\mathbf{M}_R)_{Oa} = \{26{,}1\mathbf{i} - 15{,}1\mathbf{j}\}$ lb·pés
4.54. $(M_{AB})_1 = 72{,}0$ N·m, $(M_{AB})_2 = (M_{AB})_3 = 0$
4.55. $M_x = 44{,}4$ lb·pés
4.57. $M_y = 0{,}277$ N·m
4.58. $\mathbf{M}_y = \{-78{,}4\mathbf{j}\}$ lb·pés
4.59. $M_x = 15{,}0$ lb·pés, $M_y = 4{,}00$ lb·pés,
$M_z = 36{,}0$ lb·pés

4-1. If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

Consider the three vectors; with **A** vertical.

Note obd is perpendicular to **A**.



$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}|(|\mathbf{B} + \mathbf{D}|) \sin\theta_3$

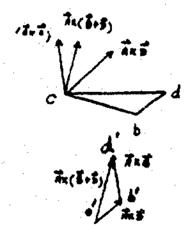
$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta_1$

$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin\theta_2$

Also, these three cross products all lie in the plane obd since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross - products also form a closed triangle $o'b'd'$ which is similar to triangle obd. Thus from the figure,

$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \mathbf{A} \times \mathbf{B} + \mathbf{A} \times \mathbf{D}$ (QED)



Note also,

$\mathbf{A} = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$

$\mathbf{B} = B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k}$

$\mathbf{D} = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

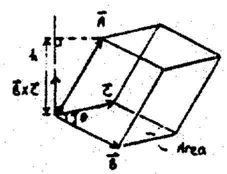
$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i} - [A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j} + [A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$

$$= [(A_yB_z - A_zB_y)\mathbf{i} - (A_xB_z - A_zB_x)\mathbf{j} + (A_xB_y - A_yB_x)\mathbf{k}] + [(A_yD_z - A_zD_y)\mathbf{i} - (A_xD_z - A_zD_x)\mathbf{j} + (A_xD_y - A_yD_x)\mathbf{k}]$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$ (QED)

4-2. Prove the triple scalar product identity $\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$.



As shown in the figure

Area $= B(C\sin\theta) = |\mathbf{B} \times \mathbf{C}|$

Thus,

Volume of parallelepiped is $|\mathbf{B} \times \mathbf{C}||h|$

But,

$$|h| = |\mathbf{A} \cdot \mathbf{u}_{(\mathbf{B} \times \mathbf{C})}| = \left|\mathbf{A} \cdot \left(\frac{\mathbf{B} \times \mathbf{C}}{|\mathbf{B} \times \mathbf{C}|}\right)\right|$$

Thus,

Volume $= |\mathbf{A} \cdot \mathbf{B} \times \mathbf{C}|$

Since $|\mathbf{A} \times \mathbf{B} \cdot \mathbf{C}|$ represents this same volume then

$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ (QED)

Also,

$LHS = \mathbf{A} \cdot \mathbf{B} \times \mathbf{C}$

$$= (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) \cdot \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}$$

$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_y - B_yC_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

$RHS = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \cdot (C_x\mathbf{i} + C_y\mathbf{j} + C_z\mathbf{k})$$

$$= C_x(A_yB_z - A_zB_y) - C_y(A_xB_z - A_zB_x) + C_z(A_xB_y - A_yB_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Thus, $LHS = RHS$

$\mathbf{A} \cdot \mathbf{B} \times \mathbf{C} = \mathbf{A} \times \mathbf{B} \cdot \mathbf{C}$ (QED)

4-3. Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = 0$, the three vectors *must* lie in the same plane.

Consider,

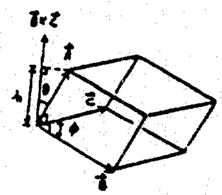
$$|\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})| = |\mathbf{A}||\mathbf{B}\times\mathbf{C}|\cos\theta$$

$$= (|\mathbf{A}|\cos\theta)|\mathbf{B}\times\mathbf{C}|$$

$$= |h||\mathbf{B}\times\mathbf{C}|$$

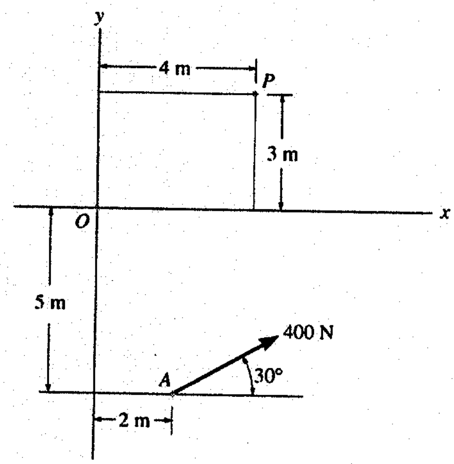
$$= BC|h|\sin\phi$$

= volume of parallelepiped.



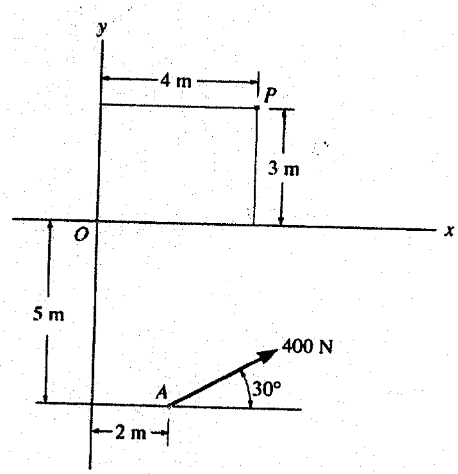
If $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = 0$, then the volume equals zero, so that **A**, **B**, and **C** are coplanar.

***4-4.** Determine the magnitude and directional sense of the moment of the force at *A* about point *O*.



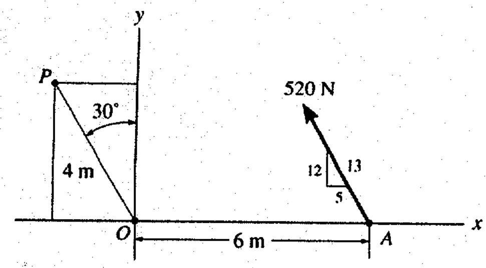
$$\circlearrowleft + \; M_O = 400\cos 30°(5) + 400\sin 30°(2)$$
$$= 2132 \text{ N}\cdot\text{m}$$
$$= 2.13 \text{ kN}\cdot\text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans}$$

4-5. Determine the magnitude and directional sense of the moment of the force at *A* about point *P*.



$$\circlearrowleft + \; M_P = 400\cos 30°(8) - 400\sin 30°(2)$$
$$= 2371 \text{ N}\cdot\text{m}$$
$$= 2.37 \text{ kN}\cdot\text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans}$$

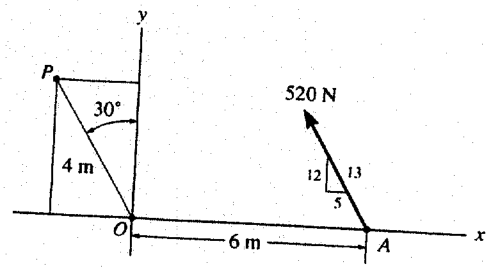
4-6. Determine the magnitude and directional sense of the moment of the force at *A* about point *O*.



$\circlearrowleft + \; M_O = 520\left(\frac{12}{13}\right)(6)$

$= 2880 \text{ N}\cdot\text{m} = 2.88 \text{ kN}\cdot\text{m}$ **(Counterclockwise)** **Ans**

4-7. Determine the magnitude and directional sense of the moment of the force at *A* about point *P*.

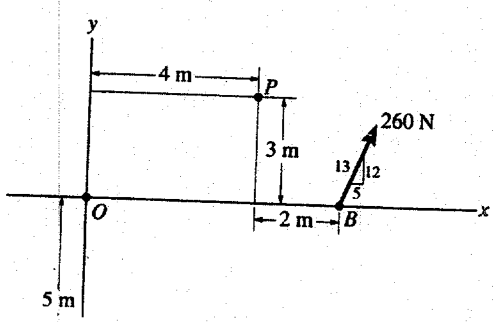


$\circlearrowleft + \; M_P = 520\left(\frac{12}{13}\right)(6 + 4\sin 30°) - 520\left(\frac{5}{13}\right)(4\cos 30°)$

$= 3147 \text{ N}\cdot\text{m}$

$= 3.15 \text{ kN}\cdot\text{m}$ **(Counterclockwise)** **Ans**

***4-8.** Determine the magnitude and directional sense of the resultant moment of the forces about point *O*.



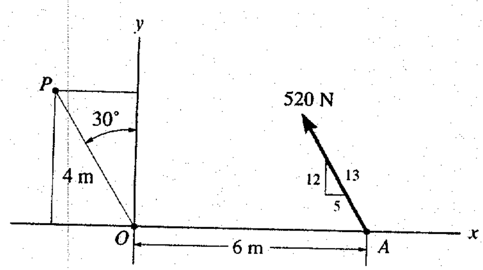
$\circlearrowleft + M_O = 400\sin 30°(2) + 400\cos 30°(5) + 260\left(\frac{12}{13}\right)(6)$

$= 3572.1 \text{ N}\cdot\text{m} = 3.57 \text{ kN}\cdot\text{m} \circlearrowleft$ **Ans**

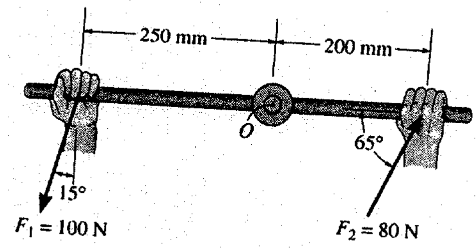
4-9. Determine the magnitude and directional sense of the resultant moment of the forces about point *P*.

$\circlearrowleft + M_P = 260\left(\frac{5}{13}\right)(3) + 260\left(\frac{12}{13}\right)(2) - 400\sin 30°(2) + 400\cos 30°(8)$

$= 3151 \text{ N}\cdot\text{m} = 3.15 \text{ kN}\cdot\text{m} \circlearrowleft$ **Ans**



4-10. The wrench is used to loosen the bolt. Determine the moment of each force about the bolt's axis passing through point O.



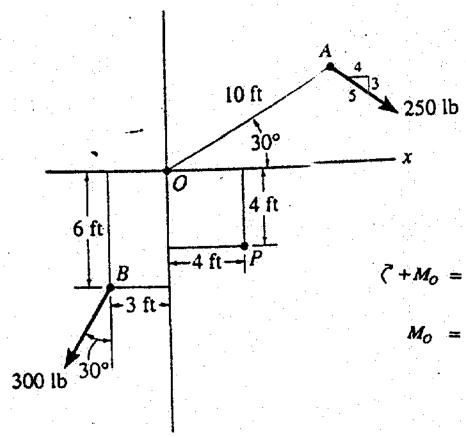
$\curvearrowleft + \left(M_{F_1}\right)_O = 100\cos 15°(0.25)$

$= 24.1\ \text{N}\cdot\text{m}$ ***(Counterclockwise)*** **Ans**

$\curvearrowleft + \left(M_{F_2}\right)_O = 80\sin 65°(0.2)$

$= 14.5\ \text{N}\cdot\text{m}$ ***(Counterclockwise)*** **Ans**

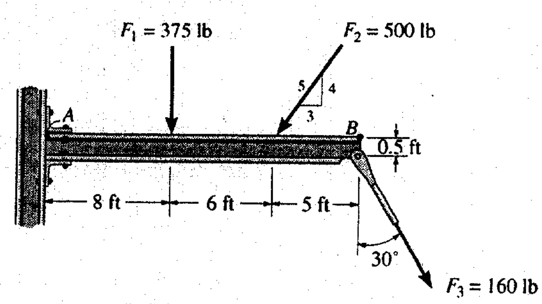
4-11. Determine the magnitude and directional sense of the resultant moment of the forces about point O.



$\curvearrowleft + M_O = 250(\frac{4}{5})(10\sin 30°) + 250(\frac{3}{5})(10\cos 30°) + 300(\sin 30°)(6) - 300(\cos 30°)(3)$

$M_O = 2419.62\ \text{lb}\cdot\text{ft} = 2.42\ \text{kip}\cdot\text{ft}$ ↺ **Ans**

***4-12.** Determine the moment about point A of each of the three forces acting on the beam.



$\curvearrowleft + \left(M_{F_1}\right)_A = -375(8)$

$= -3000\ \text{lb}\cdot\text{ft} = 3.00\ \text{kip}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

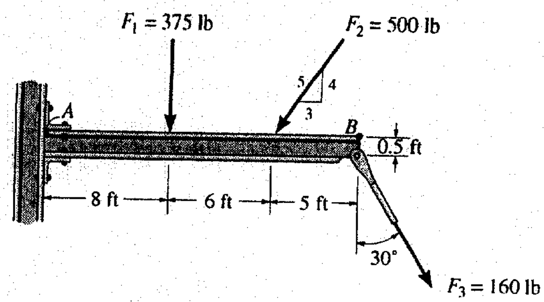
$\curvearrowleft + \left(M_{F_2}\right)_A = -500\left(\frac{4}{5}\right)(14)$

$= -5600\ \text{lb}\cdot\text{ft} = 5.60\ \text{kip}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

$\curvearrowleft + \left(M_{F_3}\right)_A = -160(\cos 30°)(19) + 160\sin 30°(0.5)$

$= -2593\ \text{lb}\cdot\text{ft} = 2.59\ \text{kip}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

***4-13.** Determine the moment about point B of each of the three forces acting on the beam.



$\circlearrowleft +\ (M_{F_1})_B = 375(11)$

$= 4125\ \text{lb}\cdot\text{ft} = 4.125\ \text{kip}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

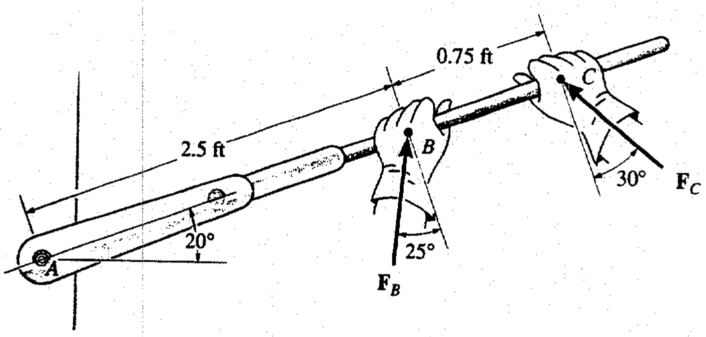
$\circlearrowleft +\ (M_{F_2})_B = 500\left(\frac{4}{5}\right)(5)$

$= 2000\ \text{lb}\cdot\text{ft} = 2.00\ \text{kip}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

$\circlearrowleft +\ (M_{F_3})_B = 160\sin 30°(0.5) - 160\cos 30°(0)$

$= 40.0\ \text{lb}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

4-14. Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



$\circlearrowleft +M_B = 40\cos 25°(2.5) = 90.6\ \text{lb}\cdot\text{ft}\ \circlearrowleft$ **Ans**

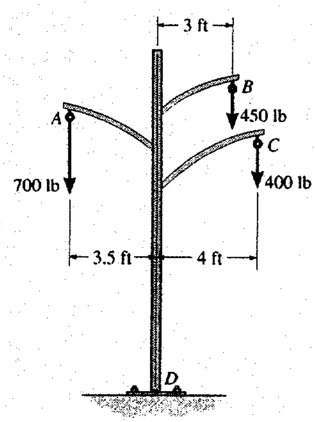
$\circlearrowleft +M_C = 50\cos 30°(3.25) = 141\ \text{lb}\cdot\text{ft}\ \circlearrowleft$ **Ans**

4-15. If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.

$\circlearrowleft +M_A = 30\cos 25°(2.5) + 45\cos 30°(3.25)$

$= 195\ \text{lb}\cdot\text{ft}\ \circlearrowleft$ **Ans**

***4-16.** The power pole supports the three lines, each line exerting a vertical force on the pole due to its weight as shown. Determine the resultant moment at the base D due to all of these forces. If it is possible for wind or ice to snap the lines, determine which line(s) when removed create(s) a condition for the greatest moment about the base. What is this resultant moment?



$\circlearrowleft + M_{R_D} = \Sigma Fd;\quad M_{R_D} = 700(3.5) - 450(3) - 400(4)$

$= -500\ \text{lb}\cdot\text{ft} = 500\ \text{lb}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

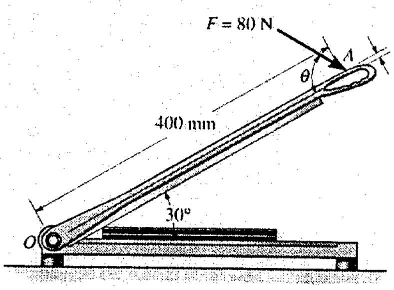
When the cable at *A* is removed it will create the greatest moment at point *D*. **Ans**

$\circlearrowleft + (M_{R_D})_{max} = \Sigma Fd;$

$(M_{R_D})_{max} = -450(3) - 400(4)$

$= -2950\ \text{lb}\cdot\text{ft} = 2.95\ \text{kip}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

4-17. A force of 80 N acts on the handle of the paper cutter at A. Determine the moment created by this force about the hinge at O, if $\theta = 60°$. At what angle θ should the force be applied so that the moment it creates about point O is a maximum (clockwise)? What is this maximum moment?



$\circlearrowleft + M_o = \Sigma Fd;\quad M_o = -80\cos\theta(0.01) - 80\sin\theta(0.4)$

$= -(0.800\cos\theta + 32.0\sin\theta)\ \text{N}\cdot\text{m}$

$= (0.800\cos\theta + 32.0\sin\theta)\ \text{N}\cdot\text{m}$ *(Clockwise)*

At $\theta = 60°$, $\quad M_o = 0.800\cos 60° + 32.0\sin 60°$

$= 28.1\ \text{N}\cdot\text{m}$ *(Clockwise)* **Ans**

In order to produce the maximum and minimum moment about point A, $\dfrac{dM_o}{d\theta} = 0$

$$\frac{dM_o}{d\theta} = 0 = -0.800\sin\theta + 32.0\cos\theta$$

$\theta = 88.568° = 88.6°$ **Ans**

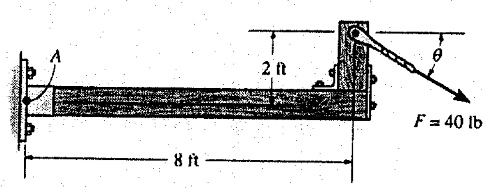
$$\frac{d^2M_A}{d\theta^2} = -0.800\cos\theta - 32.0\sin\theta$$

Since $\left.\dfrac{d^2M_A}{d\theta^2}\right|_{\theta=88.568°} = -0.800\cos 88.568° - 32.0\sin 88.568° =$ -32.00 is a negative value, indeed at $\theta = 88.568°$, the 80 N produces a maximum clockwise moment at O. This maximum clockwise moment is

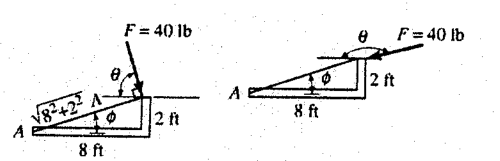
$(M_O)_{max} = 0.800\cos 88.568° + 32.0\sin 88.568°$

$= 32.0\ \text{N}\cdot\text{m}$ *(Clockwise)* **Ans**

4-18. Determine the direction $\theta(0° \le \theta \le 180°)$ of the force $F = 40$ lb so that it produces (a) the maximum moment about point A and (b) the minimum moment about point A. Compute the moment in each case.



(a) $\circlearrowleft + (M_A)_{max} = 40(\sqrt{8^2+2^2}) = 330\ \text{lb}\cdot\text{ft}$ **Ans**



$\phi = \tan^{-1}\left(\frac{2}{8}\right) = 14.04°$

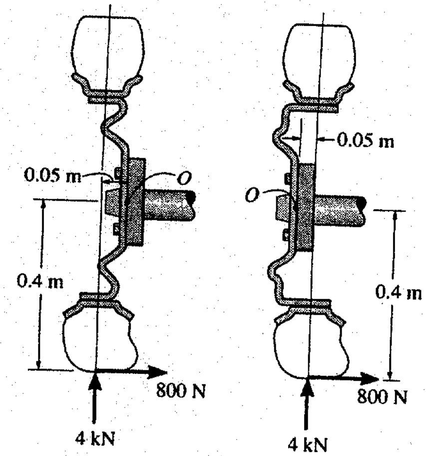
$\theta = 90° - 14.04° = 76.0°$ **Ans**

(b) $\circlearrowleft + (M_A)_{min} = 0$ **Ans**

$\phi = \tan^{-1}\left(\frac{2}{8}\right) = 14.04°$

$\theta = 180° - 14.04° = 166°$ **Ans**

***4-19.** The hub of the wheel can be attached to the axle either with negative offset (left) or with positive offset (right). If the tire is subjected to both a normal and radial load as shown, determine the resultant moment of these loads about the axle, point O for both cases.



For case 1 with negative offset, we have

$$\circlearrowleft + M_O = 800(0.4) - 4000(0.05)$$
$$= 120\ \text{N}\cdot\text{m} \quad \textbf{(Counterclockwise)} \qquad \textbf{Ans}$$

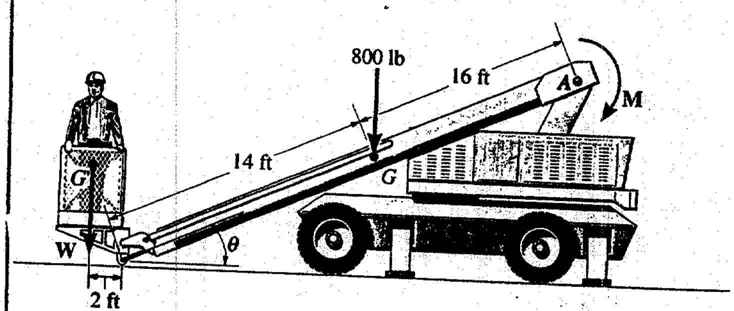
For case 2 with positive offset, we have

$$\circlearrowleft + M_O = 800(0.4) + 4000(0.05)$$
$$= 520\ \text{N}\cdot\text{m} \quad \textbf{(Counterclockwise)} \qquad \textbf{Ans}$$

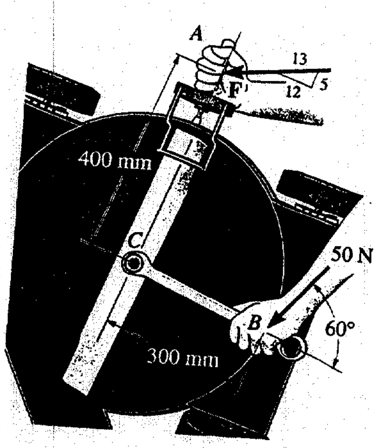
***4-20.** The boom has a length of 30 ft, a weight of 800 lb, and mass center at G. If the maximum moment that can be developed by the motor at A is $M = 20(10^3)$ lb · ft, determine the maximum load W, having a mass center at G', that can be lifted. Take $\theta = 30°$.

$$20(10^3) = 800(16\cos 30°) + W(30\cos 30° + 2)$$

$W = 319$ lb **Ans**



4-21. The tool at A is used to hold a power lawnmower blade stationary while the nut is being loosened with the wrench. If a force of 50 N is applied to the wrench at B in the direction shown, determine the moment it creates about the nut at C. What is the magnitude of force **F** at A so that it creates the opposite moment about C?



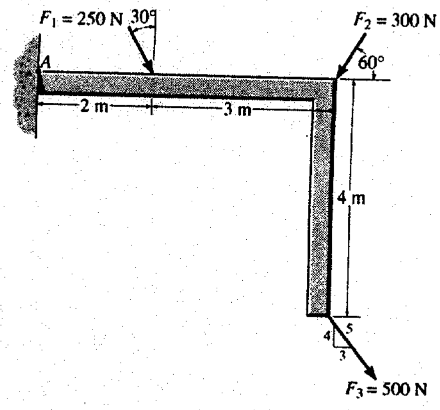
(a) $\circlearrowleft + M_A = 50\sin 60°(0.3)$

$M_A = 12.99 = 13.0\ \text{N}\cdot\text{m}$ **Ans**

(b) $\circlearrowleft + M_A = 0; \quad -12.99 + F(\frac{12}{13})(0.4) = 0$

$F = 35.2$ N **Ans**

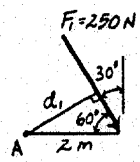
4-22. Determine the moment of each of the three forces about point A. Solve the problem first by using each force as a whole, and then by using the principle of moments.



The moment arm measured perpendicular to each force from point A is

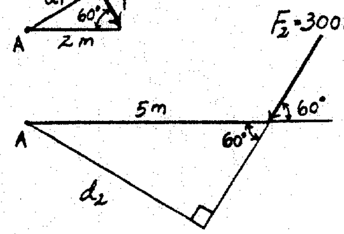
$$d_1 = 2\sin 60° = 1.732 \text{ m}$$
$$d_2 = 5\sin 60° = 4.330 \text{ m}$$
$$d_3 = 2\sin 53.13° = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have



$$\circlearrowleft + \quad (M_{F_1})_A = -250(1.732)$$
$$= -433 \text{ N}\cdot\text{m} = 433 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_2})_A = -300(4.330)$$
$$= -1299 \text{ N}\cdot\text{m} = 1.30 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

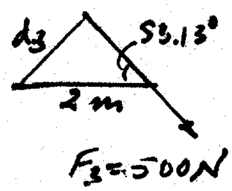


$$\circlearrowleft + \quad (M_{F_3})_A = -500(1.60)$$
$$= -800 \text{ N}\cdot\text{m} = 800 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

Using principle of moments, we have

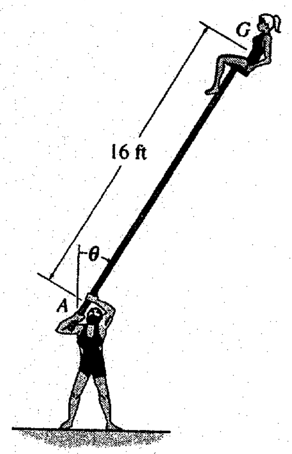
$$\circlearrowleft + \quad (M_{F_1})_A = -250\cos 30°(2)$$
$$= -433 \text{ N}\cdot\text{m} = 433 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_{F_2})_A = -300\sin 60°(5)$$
$$= -1299 \text{ N}\cdot\text{m} = 1.30 \text{ kN}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$



$$\circlearrowleft + \quad (M_{F_3})_A = 500\left(\frac{3}{5}\right)(4) - 500\left(\frac{4}{5}\right)(5)$$
$$= -800 \text{ N}\cdot\text{m} = 800 \text{ N}\cdot\text{m} \quad (\textit{Clockwise}) \qquad \textbf{Ans}$$

4-23. As part of an acrobatic stunt, a man supports a girl who has a weight of 120 lb and is seated on a chair on top of the pole. If her center of gravity is at G, and if the maximum counterclockwise moment the man can exert on the pole at A is 250 lb·ft, determine the maximum angle of tilt, θ, which will not allow the girl to fall, i.e., so her clockwise moment about A does not exceed 250 lb·ft.

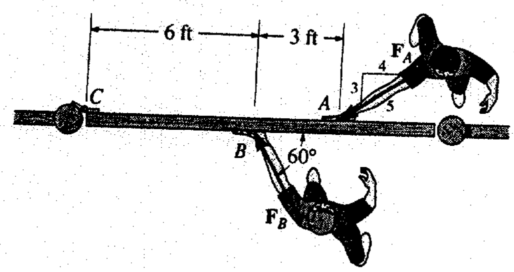


In order to prevent the girl from falling down, the clockwise moment produced by the girl's weight must not exceeded 250 lb · ft.

$$M_A = 120(16\sin\theta) \le 250$$
$$\sin\theta \le 0.1302$$

$$\theta = 7.48° \qquad \textbf{Ans}$$

4-24. The two boys push on the gate with forces of $F_A = 30$ lb and $F_B = 50$ lb as shown. Determine the moment of each force about C. Which way will the gate rotate, clockwise or counterclockwise? Neglect the thickness of the gate.



$\circlearrowleft + \ (M_{F_A})_C = -30\left(\frac{3}{5}\right)(9)$

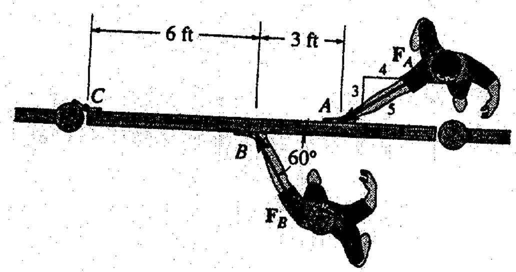
$= -162 \text{ lb}\cdot\text{ft} = 162 \text{ lb}\cdot\text{ft}$ ***(Clockwise)*** **Ans**

$\circlearrowleft + \ (M_{F_B})_C = 50(\sin 60°)(6)$

$= 260 \text{ lb}\cdot\text{ft}$ ***(Counterclockwise)*** **Ans**

Since $(M_{F_B})_C > (M_{F_A})_C$, the gate will rotate ***Counterclockwise.*** **Ans**

4-25. Two boys push on the gate as shown. If the boy at B exerts a force of $F_B = 30$ lb, determine the magnitude of the force F_A the boy at A must exert in order to prevent the gate from turning. Neglect the thickness of the gate.

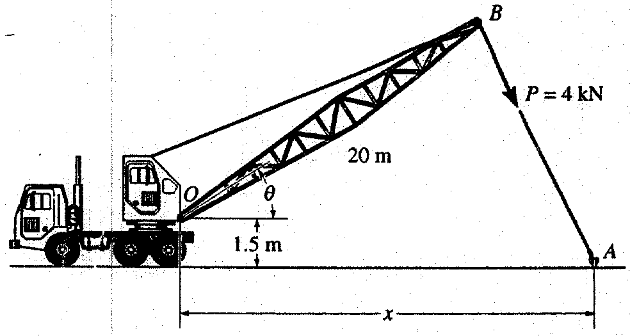


In order to prevent the gate from turning, the resultant moment about point C must be equal to zero.

$+ \ M_{R_C} = \Sigma Fd; \quad M_{R_C} = 0 = 30\sin 60°(6) - F_A\left(\frac{3}{5}\right)(9)$

$F_A = 28.9 \text{ lb}$ **Ans**

4-26. The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at A so that this force creates a maximum moment about point O. What is this moment?



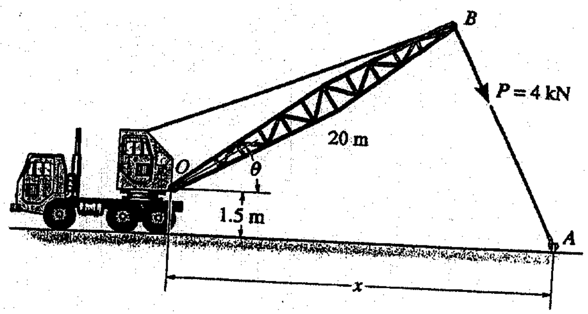
Maximum moment, $OB \perp BA$

$\circlearrowleft + (M_O)_{max} = 4000(20) = 80$ kN·m **Ans**

$4\text{ kN} \sin 60°(x) - 4\text{ kN} \cos 60°(1.5) = 80$ kN·m

$x = 24.0$ m **Ans**

4-27. The towline exerts a force of $P = 4$ kN at the end of the 20-m-long crane boom. If $x = 25$ m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?



Maximum moment, $OB \perp BA$

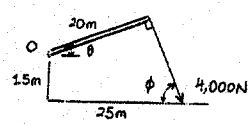
$\circlearrowleft + (M_O)_{max} = 4000(20) = 80\,000$ N·m $= 80.0$ kN·m **Ans**

$4000 \sin\phi(25) - 4000 \cos\phi(1.5) = 80\,000$

$25 \sin\phi - 1.5 \cos\phi = 20$

$\phi = 56.43°$

$\theta = 90° - 56.43° = 33.6°$ **Ans**



Also,

$(1.5)^2 + z^2 = y^2$

$2.25 + z^2 = y^2$

Similar triangles

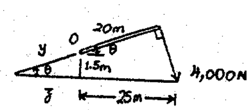
$$\frac{20+y}{z} = \frac{25+z}{y}$$

$20y + y^2 = 25z + z^2$

$20(\sqrt{2.25 + z^2}) + 2.25 + z^2 = 25z + z^2$

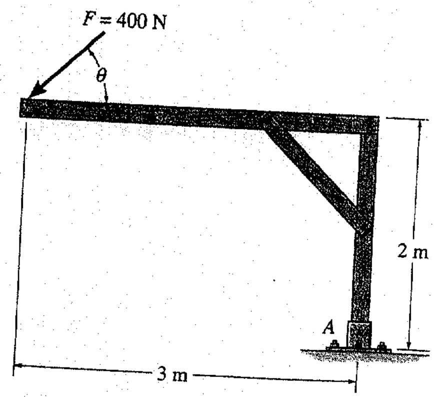
$z = 2.259$ m

$y = 2.712$ m



$\theta = \cos^{-1}\left(\frac{2.259}{2.712}\right) = 33.6°$ **Ans**

***4-28.** Determine the direction θ for $0° \leq \theta \leq 180°$ of the force **F** so that **F** produces (a) the maximum moment about point A and (b) the minimum moment about point A. Calculate the moment in each case.



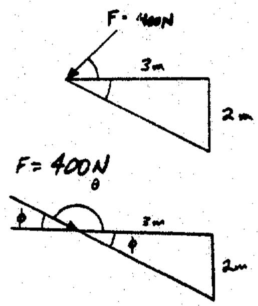
a)

$\circlearrowleft + M_A = 400\sqrt{(3)^2 + (2)^2} = 1442 \text{ N}\cdot\text{m}$

$M_A = 1.44 \text{ kN}\cdot\text{m}$ **Ans**

$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$

$\theta = 90° - 33.69° = 56.3°$ **Ans**



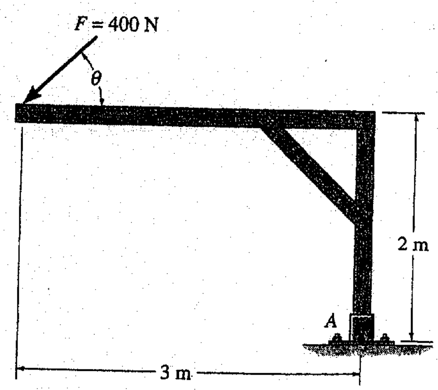
b)

$\circlearrowleft + M_A = 0$ **Ans**

$\phi = \tan^{-1}\left(\frac{2}{3}\right) = 33.69°$

$\theta = 180° - 33.69° = 146°$ **Ans**

4-29. Determine the moment of the force **F** about point A as a function of θ. Plot the results of M (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 180°$.



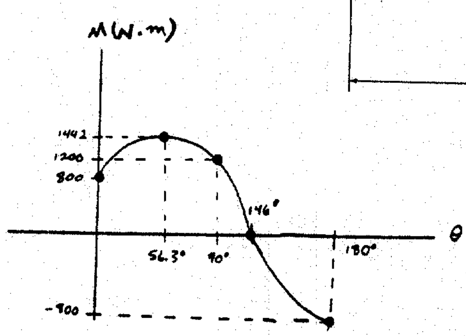
$\circlearrowleft + M_A = 400 \sin\theta(3) + 400 \cos\theta(2)$

$= 1200 \sin\theta + 800 \cos\theta$ **Ans**

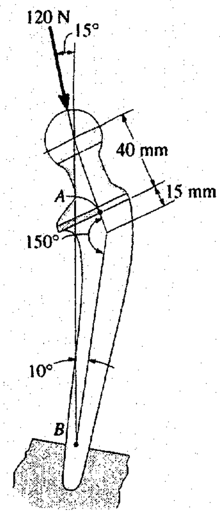
$\frac{dM_A}{d\theta} = 1200 \cos\theta - 800 \sin\theta = 0$

$\theta = \tan^{-1}\left(\frac{1200}{800}\right) = 56.3°$

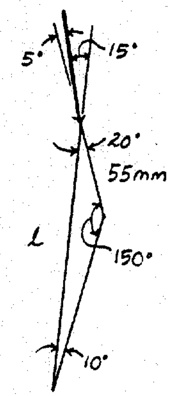
$(M_A)_{max} = 1200 \sin 56.3° + 800 \cos 56.3° = 1442 \text{ N}\cdot\text{m}$



4-30. The total hip replacement is subjected to a force of $F = 120$ N. Determine the moment of this force about the neck at A and at the stem B.



Moment About Point A : The angle between the line of action of the load and the neck axis is $20° - 15° = 5°$.



$$\circlearrowleft + M_A = 120\sin 5°(0.04)$$
$$= 0.418 \text{ N}\cdot\text{m} \quad \textit{(Counterclockwise)} \qquad \textbf{Ans}$$

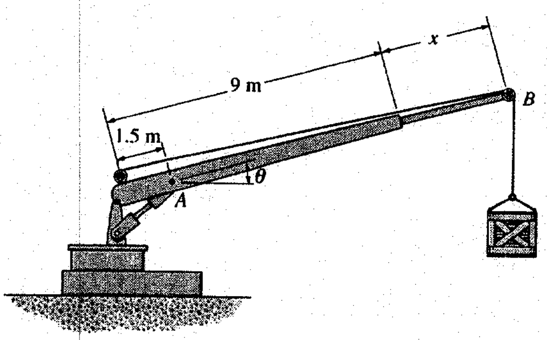
Moment About Point B : The dimension l can be determined using the law of sines.

$$\frac{l}{\sin 150°} = \frac{55}{\sin 10°} \qquad l = 158.4 \text{ mm} = 0.1584 \text{ m}$$

Then,

$$+ M_B = -120\sin 15°(0.1584)$$
$$= -4.92 \text{ N}\cdot\text{m} = 4.92 \text{ N}\cdot\text{m} \quad \textit{(Clockwise)} \qquad \textbf{Ans}$$

***4-31.** The crane can be adjusted for any angle $0° \le \theta \le 90°$ and any extension $0 \le x \le 5$ m. For a suspended mass of 120 kg, determine the moment developed at A as a function of x and θ. What values of both x and θ develop the maximum possible moment at A? Compute this moment. Neglect the size of the pulley at B.

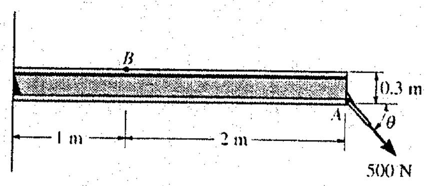


$$\circlearrowleft + M_A = -120(9.81)(7.5+x)\cos\theta$$
$$= \{-1177.2\cos\theta(7.5+x)\} \text{ N}\cdot\text{m}$$
$$= \{1.18\cos\theta(7.5+x)\} \text{ kN}\cdot\text{m} \quad \textit{(clockwise)} \qquad \textbf{Ans}$$

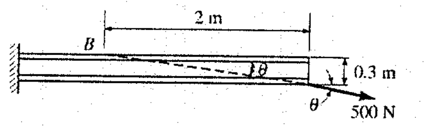
The maximum moment at A occurs when $\theta = 0°$ and $x = 5$ m. **Ans**

$$\circlearrowleft + (M_A)_{max} = \{-1177.2\cos 0°(7.5+5)\} \text{ N}\cdot\text{m}$$
$$= -14715 \text{ N}\cdot\text{m}$$
$$= 14.7 \text{ kN}\cdot\text{m} \quad \textit{(clockwise)} \qquad \textbf{Ans}$$

***4-32.** Determine the angle θ at which the 500-N force must act at A so that the moment of this force about point B is equal to zero.



This problem requires that the resultant moment about point B be equal to zero.



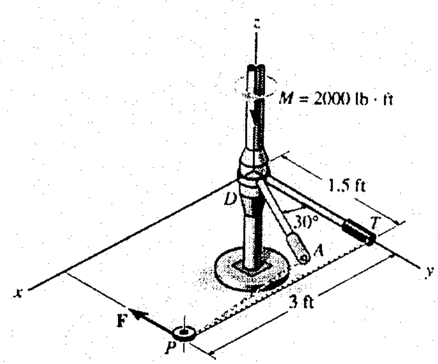
$\circlearrowleft +M_{R_t} = \Sigma Fd;\quad M_{R_t} = 0 = 500\cos\theta(0.3) - 500\sin\theta(2)$

$\theta = 8.53°$ **Ans**

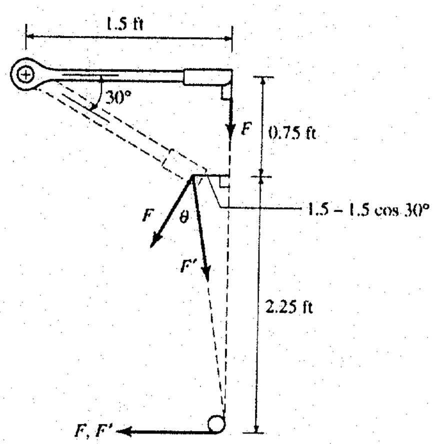
Also note that if the line of action of the 500 N force passes through point B, it produces zero moment about point B. Hence, from the geometry

$$\theta = \tan^{-1}\left(\frac{0.3}{2}\right) = 8.53°$$

4-33. Segments of drill pipe D for an oil well are tightened a prescribed amount by using a set of tongs T, which grip the pipe, and a hydraulic cylinder (not shown) to regulate the force F applied to the tongs. This force acts along the cable which passes around the small pulley P. If the cable is originally perpendicular to the tongs as shown, determine the magnitude of force F which must be applied so that the moment about the pipe is $M = 2000$ lb · ft. In order to maintain this same moment what magnitude of **F** is required when the tongs rotate 30° to the dashed position? *Note*: The angle DAP is not 90° in this position.



This problem requires that the moment produced by **F** and **F′** about the z axis is 2000 lb · ft.



$M_z = 2000 = F(1.5)$

$F = 1333.3$ lb $= 1.33$ kip **Ans**

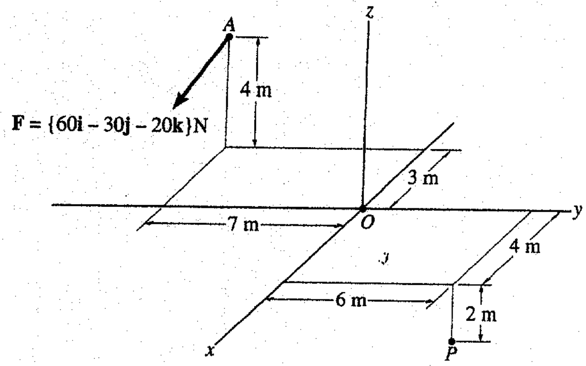
$F = F'\cos\theta$, where

$$\theta = 30° + \tan^{-1}\left(\frac{1.5 - 1.5\cos 30°}{2.25}\right)$$

$= 35.104°$

$$F' = \frac{1333.33}{\cos 35.104°} = 1.63 \text{ kip}$$ **Ans**

4-34. Determine the moment of the force at A about point O. Express the result as a Cartesian vector.



Position Vector:

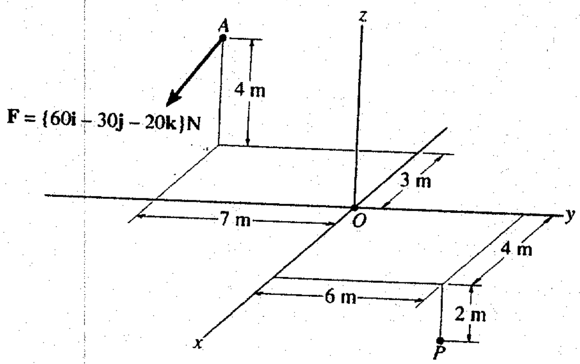
$$\mathbf{r}_{OA} = \{(-3-0)\mathbf{i} + (-7-0)\mathbf{j} + (4-0)\mathbf{k}\}\text{ m}$$
$$= \{-3\mathbf{i} - 7\mathbf{j} + 4\mathbf{k}\}\text{ m}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -7 & 4 \\ 60 & -30 & -20 \end{vmatrix}$$

$$= \{260\mathbf{i} + 180\mathbf{j} + 510\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans**

4-35. Determine the moment of the force at A about point P. Express the result as a Cartesian vector.



Position Vector:

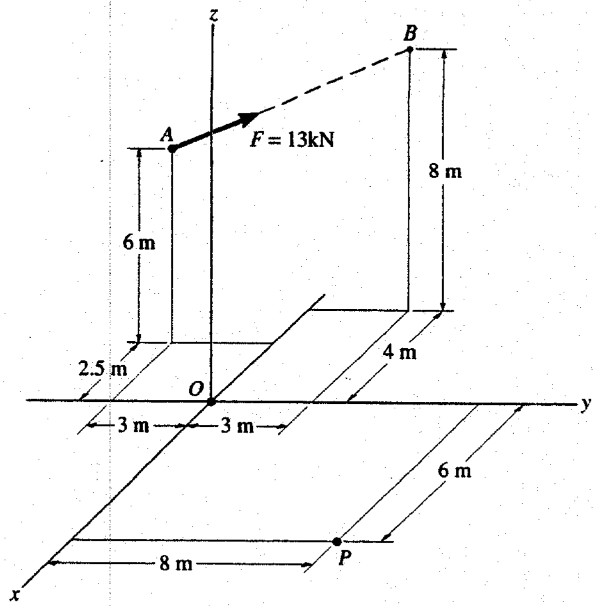
$$\mathbf{r}_{PA} = \{(-3-4)\mathbf{i} + (-7-6)\mathbf{j} + [4-(-2)]\mathbf{k}\}\text{ m}$$
$$= \{-7\mathbf{i} - 13\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -7 & -13 & 6 \\ 60 & -30 & -20 \end{vmatrix}$$

$$= \{440\mathbf{i} + 220\mathbf{j} + 990\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans**

***4-36.** Determine the moment of the force **F** at A about point O. Express the result as a Cartesian vector.



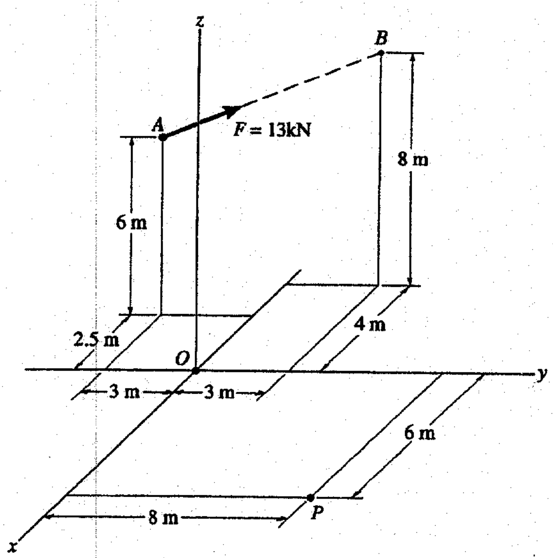
$\mathbf{r}_{AB} = \{-1.5\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\}$ m

$r_{AB} = \sqrt{(-1.5)^2 + 6^2 + 2^2} = 6.5$ m

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -2.5 & -3 & 6 \\ -\frac{1.5}{6.5}(13) & \frac{6}{6.5}(13) & \frac{2}{6.5}(13) \end{vmatrix}$$

$\mathbf{M}_O = \{-84\mathbf{i} - 8\mathbf{j} - 39\mathbf{k}\}$ kN·m **Ans**

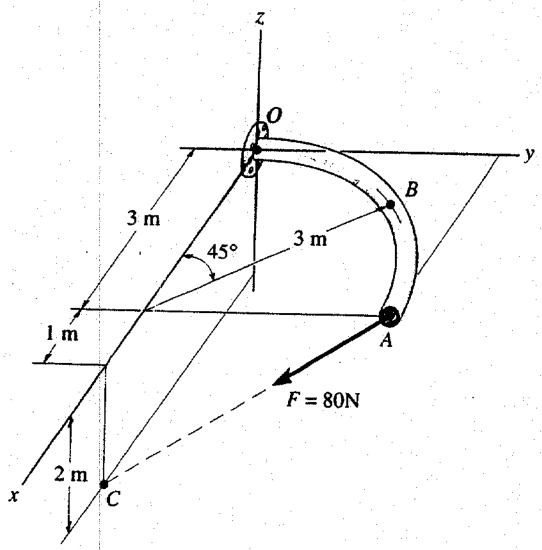
4-37. Determine the moment of the force **F** at A about point P. Express the result as a Cartesian vector.



$$\mathbf{M}_P = \mathbf{r}_{PA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -8.5 & -11 & 6 \\ -\frac{1.5}{6.5}(13) & \frac{6}{6.5}(13) & \frac{2}{6.5}(13) \end{vmatrix}$$

$\mathbf{M}_P = \{-116\mathbf{i} + 16\mathbf{j} - 135\mathbf{k}\}$ kN·m **Ans**

4-38. The curved rod lies in the x-y plane and has a radius of 3 m. If a force of $F = 80$ N acts at its end as shown, determine the moment of this force about point O.



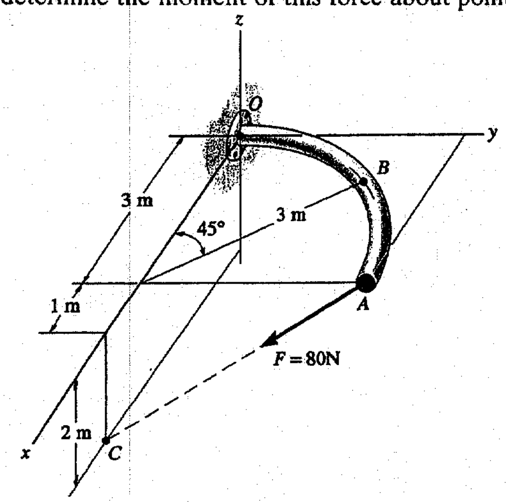
$$\mathbf{r}_{AC} = \{1\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742\ \text{m}$$

$$\mathbf{M}_O = \mathbf{r}_{OC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & -2 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$\mathbf{M}_O = \{-128\mathbf{i} + 128\mathbf{j} - 257\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4-39. The curved rod lies in the x-y plane and has a radius of 3 m. If a force of $F = 80$ N acts at its end as shown, determine the moment of this force about point B.



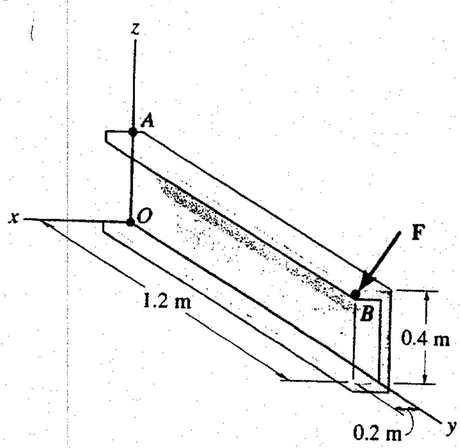
$$\mathbf{r}_{AC} = \{1\mathbf{i} - 3\mathbf{j} - 2\mathbf{k}\}\ \text{m}$$

$$r_{AC} = \sqrt{(1)^2 + (-3)^2 + (-2)^2} = 3.742\ \text{m}$$

$$\mathbf{M}_B = \mathbf{r}_{BA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3\cos 45^\circ & (3 - 3\sin 45^\circ) & 0 \\ \frac{1}{3.742}(80) & -\frac{3}{3.742}(80) & -\frac{2}{3.742}(80) \end{vmatrix}$$

$$\mathbf{M}_B = \{-37.6\mathbf{i} + 90.7\mathbf{j} - 155\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

***4-40.** The force $\mathbf{F} = \{600\mathbf{i} + 300\mathbf{j} - 600\mathbf{k}\}$ N acts at the end of the beam. Determine the moment of the force about point A.

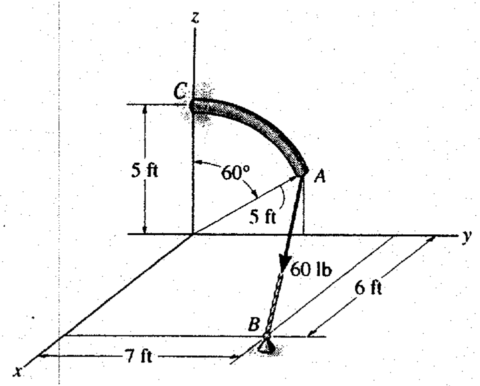


$$\mathbf{r} = \{0.2\mathbf{i} + 1.2\mathbf{j}\}\ \text{m}$$

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.2 & 1.2 & 0 \\ 600 & 300 & -600 \end{vmatrix}$$

$$\mathbf{M}_A = \{-720\mathbf{i} + 120\mathbf{j} - 660\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4-41. The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C.



Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5\sin 60° - 0)\mathbf{j} + (5\cos 60° - 5)\mathbf{k}\}\ \text{m}$$
$$= \{4.330\mathbf{j} - 2.50\mathbf{k}\}\ \text{m}$$

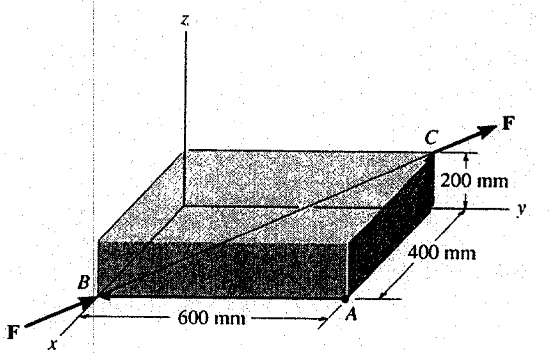
$$\mathbf{F}_{AB} = 60\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right)\ \text{lb}$$
$$= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\}\ \text{lb}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C: Applying Eq. 4-7, we have

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 51.231 & 22.797 & -21.346 \end{vmatrix}$$

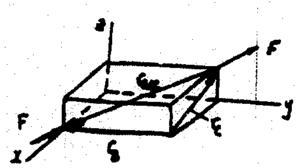
$$= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\}\ \text{lb}\cdot\text{ft}$$ **Ans**

4-42. A force $\mathbf{F}$ having a magnitude of $F = 100$ N acts along the diagonal of the parallelepiped. Determine the moment of $\mathbf{F}$ about point A, using $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ and $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$.



$$\mathbf{F} = 100\left(\frac{-0.4\mathbf{i} + 0.6\mathbf{j} + 0.2\mathbf{k}}{0.7483}\right)$$

$$\mathbf{F} = \{-53.5\mathbf{i} + 80.2\mathbf{j} + 26.7\mathbf{k}\}\ \text{N}$$

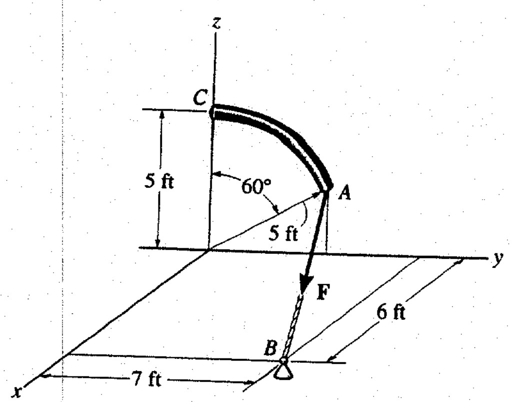


$$\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.6 & 0 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\mathbf{i} - 32.1\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans**

Also,

$$\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4 & 0 & 0.2 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\mathbf{i} - 32.1\mathbf{k}\}\ \text{N}\cdot\text{m}$$ **Ans**

4-43. Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft, to fail at the support C. This requires a moment of $M = 80$ lb · ft to be developed at C.



$$\mathbf{r}_{CA} = \{4.3301\mathbf{j} - 2.5\mathbf{k}\}\ \text{ft}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{6\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} - 5\cos 60°\mathbf{k}}{\sqrt{(6)^2 + (7 - 5\sin 60°)^2 + (-5\cos 60°)^2}}\right)$$

$$\mathbf{F}_{AB} = F_{AB}(0.8538\mathbf{i} + 0.3799\mathbf{j} - 0.3558\mathbf{k})$$

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

$$\mathbf{M}_C = F_{AB}\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.3301 & -2.5 \\ 0.8538 & 0.3799 & -0.3558 \end{vmatrix}$$

$$\mathbf{M}_C = F_{AB}(-0.5909\mathbf{i} + 2.135\mathbf{j} - 3.697\mathbf{k})$$

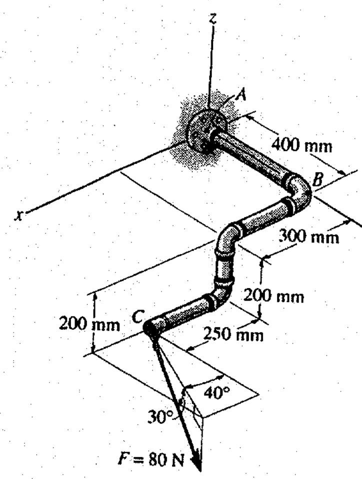
$$M_C = F_{AB}\sqrt{(-0.5909)^2 + (2.135)^2 + (-3.697)^2}$$

$$80 = F_{AB}(4.310)$$

$$F_{AB} = \frac{80}{4.310} = 18.5618\ \text{lb}$$

$$F_{AB} = 18.6\ \text{lb}$$ **Ans**

***4-44.** The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



Position Vector And Force Vector :

$$\mathbf{r}_{AC} = \{(0.55-0)\mathbf{i}+(0.4-0)\mathbf{j}+(-0.2-0)\mathbf{k}\}\ \text{m}$$
$$= \{0.55\mathbf{i}+0.4\mathbf{j}-0.2\mathbf{k}\}\ \text{m}$$

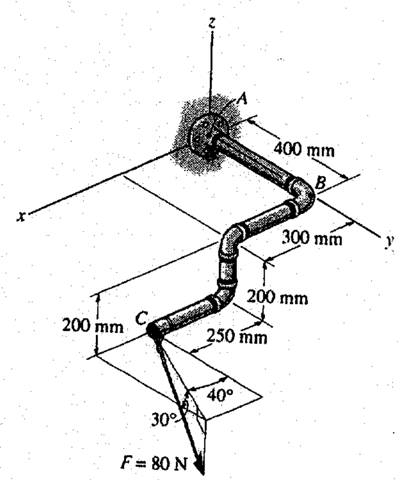
$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\ \text{N}$$
$$= \{44.53\mathbf{i}+53.07\mathbf{j}-40.0\mathbf{k}\}\ \text{N}$$

Moment of Force F About Point A : Applying Eq. 4-7, we have

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i}+13.1\mathbf{j}+11.4\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4-45. The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.



Position Vector And Force Vector :

$$\mathbf{r}_{BC} = \{(0.55-0)\mathbf{i}+(0.4-0.4)\mathbf{j}+(-0.2-0)\mathbf{k}\}\ \text{m}$$
$$= \{0.55\mathbf{i}-0.2\mathbf{k}\}\ \text{m}$$

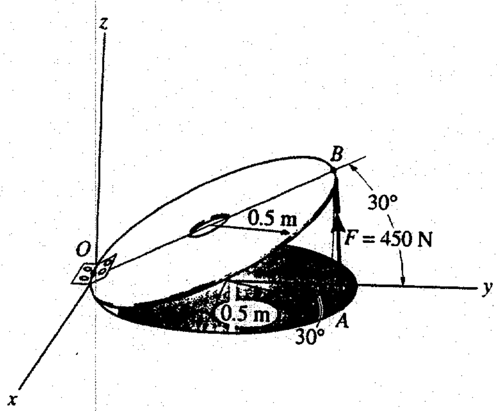
$$\mathbf{F} = 80(\cos 30^\circ \sin 40^\circ \mathbf{i} + \cos 30^\circ \cos 40^\circ \mathbf{j} - \sin 30^\circ \mathbf{k})\ \text{N}$$
$$= \{44.53\mathbf{i}+53.07\mathbf{j}-40.0\mathbf{k}\}\ \text{N}$$

Moment of Force F About Point B : Applying Eq. 4-7, we have

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i}+13.1\mathbf{j}+29.2\mathbf{k}\}\ \text{N}\cdot\text{m} \qquad \textbf{Ans}$$

4-46. Strut AB of the 1-m-diameter hatch door exerts a force of 450 N on point B. Determine the moment of this force about point O.



Position Vector And Force Vector :

$$\mathbf{r}_{OB} = \{(0-0)\mathbf{i} + (1\cos 30° - 0)\mathbf{j} + (1\sin 30° - 0)\mathbf{k}\} \text{ m} = \{0.8660\mathbf{j} + 0.5\mathbf{k}\} \text{ m}$$

$$\mathbf{r}_{OA} = \{(0.5\sin 30° - 0)\mathbf{i} + (0.5 + 0.5\cos 30° - 0)\mathbf{j} + (0-0)\mathbf{k}\} \text{ m} = \{0.250\mathbf{i} + 0.9330\mathbf{j}\} \text{ m}$$

$$\mathbf{F} = 450\left(\frac{(0-0.5\sin 30°)\mathbf{i} + [1\cos 30° - (0.5+0.5\cos 30°)]\mathbf{j} + (1\sin 30° - 0)\mathbf{k}}{\sqrt{(0-0.5\sin 30°)^2 + [1\cos 30° - (0.5+0.5\cos 30°)]^2 + (1\sin 30° - 0)^2}}\right) \text{N}$$
$$= \{-199.82\mathbf{i} - 53.54\mathbf{j} + 399.63\mathbf{k}\} \text{ N}$$

Moment of Force* F *About Point O : Applying Eq. 4-7, we have

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.8660 & 0.5 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

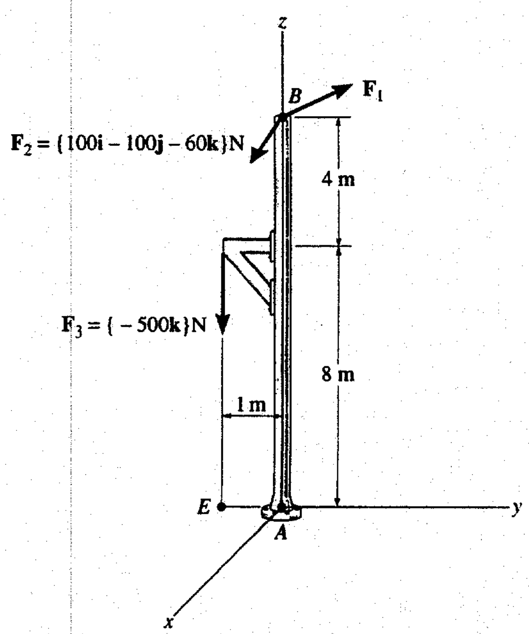
$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N} \cdot \text{m} \qquad \textbf{Ans}$$

Or

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.250 & 0.9330 & 0 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\} \text{ N} \cdot \text{m}$$

4-47. Using Cartesian vector analysis, determine the resultant moment of the three forces about the base of the column at A. Take $\mathbf{F}_1 = \{400\mathbf{i} + 300\mathbf{j} + 120\mathbf{k}\}$ N.



$$(\mathbf{M}_A)_1 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 400 & 300 & 120 \end{vmatrix} = \{-3.6\mathbf{i} + 4.8\mathbf{j}\} \text{ kN}\cdot\text{m}$$

$$(\mathbf{M}_A)_2 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & 12 \\ 100 & -100 & -60 \end{vmatrix} = \{1.2\mathbf{i} + 1.2\mathbf{j}\} \text{ kN}\cdot\text{m}$$

$$(\mathbf{M}_A)_3 = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -1 & 0 \\ 0 & 0 & -500 \end{vmatrix} = \{0.5\mathbf{i}\} \text{ kN}\cdot\text{m}$$

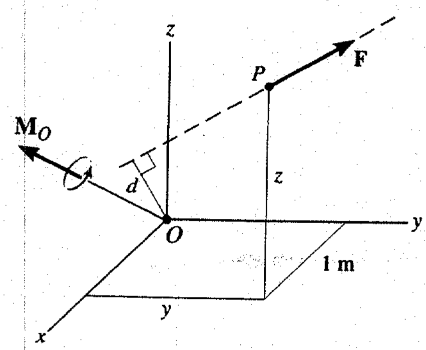
$$M_{Ax} = -3.6 + 1.2 + 0.5 = -1.90 \text{ kN}\cdot\text{m}$$

$$M_{Ay} = 4.8 + 1.2 = 6.00 \text{ kN}\cdot\text{m}$$

$$M_{Az} = 0$$

$$\mathbf{M}_R = \{-1.90\mathbf{i} + 6.00\mathbf{j}\} \text{ kN}\cdot\text{m} \qquad \textbf{Ans}$$

***4-48.** A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates.



$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$4 = y + 2z$

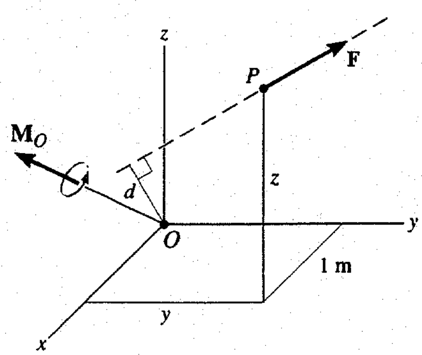
$5 = -1 + 6z$

$-14 = -2 - 6y$

$y = 2$ m **Ans**

$z = 1$ m **Ans**

4-49. The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point O of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$ N·m. If the force passes through a point having an x coordinate of 1 m, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of **F**.



$$-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$-14 = 10y - 8z$

$8 = -10 + 6z$

$2 = 8 - 6y$

$y = 1$ m **Ans**

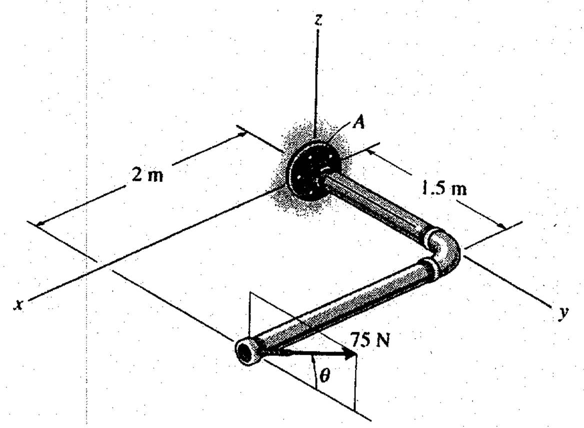
$z = 3$ m **Ans**

$M_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25$ N·m

$F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14$ N

$d = \dfrac{16.25}{14.14} = 1.15$ m **Ans**

***4-50.** Using a ring collar the 75-N force can act in the vertical plane at various angles θ. Determine the magnitude of the moment it produces about point A, plot the result of M (ordinate) versus θ (abscissa) for $0° \le \theta \le 180°$, and specify the angles that give the maximum and minimum moment.



$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.5 & 0 \\ 0 & 75\cos\theta & 75\sin\theta \end{vmatrix}$$

$$= 112.5\sin\theta\,\mathbf{i} - 150\sin\theta\,\mathbf{j} + 150\cos\theta\,\mathbf{k}$$

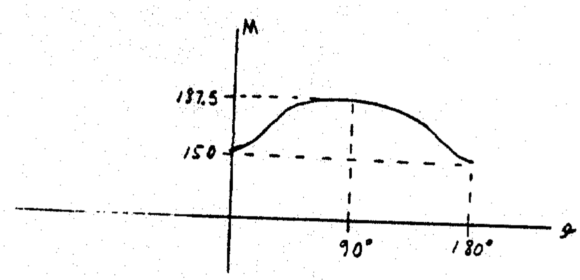
$$M_A = \sqrt{(112.5\sin\theta)^2 + (-150\sin\theta)^2 + (150\cos\theta)^2} = \sqrt{12\,656.25\sin^2\theta + 22\,500}$$

$$\frac{dM_A}{d\theta} = \frac{1}{2}(12\,656.25\sin^2\theta + 22\,500)^{-\frac{1}{2}}(12\,656.25)(2\sin\theta\cos\theta) = 0$$

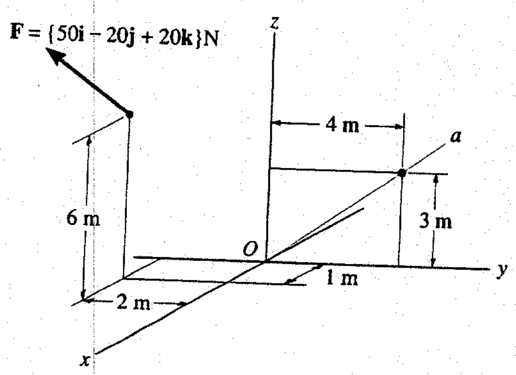
$\sin\theta\cos\theta = 0;\quad \theta = 0°,\ 90°,\ 180°$ **Ans**

$M_{max} = 187.5$ N · m at $\theta = 90°$

$M_{min} = 150$ N · m at $\theta = 0°,\ 180°$



4-51. Determine the moment of the force **F** about the Oa axis. Express the result as a Cartesian vector.



$$\mathbf{u}_{Oa} = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

$$(M_{Oa})_P = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 1 & -2 & 6 \\ 50 & -20 & 20 \end{vmatrix} = 272 \text{ N·m}$$

$$(\mathbf{M}_{Oa})_P = (M_{Oa})_P\mathbf{u}_{Oa}$$

$$= 272(\frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k})$$

$(\mathbf{M}_{Oa})_P = \{218\mathbf{j} + 163\mathbf{k}\}$ N·m **Ans**